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STEADY STATE RESPONSE OF A SECOND ORDER  
SERVOMECHANISM WITH BACKLASH AND  
RESILIENCE IN THE GEARS BETWEEN  
MOTOR AND LOAD

NORRIS O. ANDERSON, JR.  
and  
THOMAS W. LUCKETT

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STEADY STATE RESPONSE  
OF A  
SECOND ORDER SERVOMECHANISM  
WITH  
BACKLASH AND RESILIENCE IN THE  
GEARS BETWEEN MOTOR AND LOAD

\* \* \* \* \*

Norris O. Anderson, Jr.

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by

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Submitted in partial fulfillment of  
the requirements for the degree of

MASTER OF SCIENCE  
IN  
ELECTRICAL ENGINEERING

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Monterey, California

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## ABSTRACT

This thesis is concerned with the steady state response to a step input of a non-linear servomechanism having viscous friction and a load position unity feedback loop. A gear train with backlash between resilient gear teeth was located between the motor and load.

The physical equations describing the system are presented and adapted to a form for the Control Data Corporation 1604 digital computer. The computer program used in this analysis is presented along with its flow diagram and description of its operation.

The results of this thesis are presented in several forms using the parameters of system damping coefficient, distribution of friction and inertia, backlash angle and coefficient of restitution. The results are applied to sample problems in illustration of their use for design and analysis.

The authors wish to express their appreciation to Dr. George J. Thaler of the Department of Electrical Engineering, and to Dr. William Wainwright and Mr. Edward N. Ward of the Department of Mathematics for their assistance and encouragement in completing this work.

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# TABLE OF SYMBOLS

$\Theta_R$	(ONE)*	Angular step input (radians). Desired load displacement at steady state
$\Theta_C$	(THETAL)	Output displacement of load (radians).
$\dot{\Theta}_C$	(THETADL)	Output velocity of load ( $\frac{\text{radians}}{\text{sec}}$ )
$E$		Error in radians. $E = \Theta_R - \Theta_C$
$K$	(KMOTCONST)	Constant of proportionality relating position error to torque developed by the motor.
$T_m$		Developed torque of motor.
$T_L$		Torque of load.
$T_{Lm}$		Torque of load referred to motor.
$\rho$	(RHO)	$\frac{\text{Number of teeth on gear 1}}{\text{Number of teeth on gear 2}}$
$J_m$	(MOINERT)	Inertia of motor measured at motor.
$J_L$	(JLOADVAL)	Inertia of load measured at load.
$J$		Total inertial measured at motor $J = J_m + \rho^2 J_L$
$F_m$	(FMOTOR)	Friction of motor measured at motor.
$F_L$	(FLOAD)	Friction of load measured at load.
$\Delta$	(DELTA)	Backlash of gearbox measured in radians at the output.
$N_S$		Slope of phase trajectory (combined system). $N_S \triangleq \frac{d\dot{\Theta}_C}{d\Theta_C} = \frac{\ddot{\Theta}_C}{\dot{\Theta}_C}$
$N_L$		Slope of phase trajectory (load floating free). $N_L \triangleq \frac{d\dot{\Theta}_C}{d\Theta_C} = \frac{\ddot{\Theta}_C}{\dot{\Theta}_C}$
$\dot{\Theta}_m, \dot{\Theta}_C$		Velocities of motor and load after impact.

\*( ) Terms indicate computer mnemonics referred to in Sec. 3, Computer Program Development.

$e$	(RESTITUT)	Coefficient of restitution.
$\xi$	(ZETA)	System damping coefficient.
$\omega_n$	(OMEGAN)	System natural frequency.
$\omega_n^2$	(OMEGANSQ)	System natural frequency, squared.
$\frac{f_L}{F_T}$	(FLFTPRIN)	Load friction ratio with respect to friction of the system.
$\frac{J_m}{J_L}$	(INERTRAT)	Inertia ratio.
$\Theta_m$	(THETAM)	Displacement of motor (radians).
$\dot{\Theta}_m$	(THETADM)	Velocity of motor $\left(\frac{\text{radians}}{\text{sec}}\right)$



## 1. Introduction and Background

The problem under investigation was the response to a unit step input of a nonlinear servomechanism having viscous friction and load-position unity feedback loop. A gear train with backlash between the resilient gear teeth was located between the motor and load. A determination of the possible existence of a limit cycle with variation of several parameters was made. The variable parameters considered were motor, load, and system time constants, backlash and the coefficient of restitution of the gear teeth. If a limit cycle existed, a further investigation was made into the change of the size of the limit cycle with variation of the above parameters.

If a gear train is required between the motor and load in a servomechanism, the gear train may be treated as ideal with perfect meshing of the gears. In this case the components of the system are joined at all times and the system may be described by a single linear differential equation. The classical linear solution in either the frequency or time domain is the result.

Present production methods cannot meet the requirements of an ideal gear train, having perfect engagement of the teeth, nor is the ideal gear train desirable from the standpoint of wear on the gear faces. The result is that the practical gear train has separation of the gear teeth when the velocities of the motor and load are different. The response of a servomechanism with backlash is in effect discontinuous or nonlinear. The net system acts as three individual but dependent systems with the boundary conditions of each being specified by certain physical laws.

Two accepted methods of analysis are available for solution of the

problem. The first is the solution by describing function methods while the second is by analytical methods. Describing function methods are primarily frequency response techniques and depend upon the assumption that the nonlinear element may be considered as linear over the range of consideration. A further assumption is made that the input to the nonlinear element is a pure sinusoid with no harmonic frequencies. The describing function method is primarily concerned with steady state results.

The analytical solution, which is the second method of considering the problem is primarily a transient response technique concerned with solution in the time domain. This approach also requires certain assumptions concerning the behavior of the system. The assumptions used in several investigations are pointed out in the following paragraphs, while the assumptions of this investigation are outlined in Section 3, Development of Equations. The validity of the analysis by any method depends on the accuracy with which the real system has been defined.

Chestnut and Mayer, Ref. a, have considered the backlash problem using describing functions and supported the findings by analog computer studies. The main difference between this cited work and previous describing function analyses was the simulation of backlash by a dead zone between the motor and load and springiness in the interconnecting shaft.

Various analytical approaches have been made to the problem of backlash, Lutkenhouse, Ref. b, used graphical methods in the phase plane to solve several cases of plastic impact between the gear teeth of a second order servomechanism. Pastel and Thaler, Ref. c, developed



analytical equations for the existence of limit cycles using phase plane equations for the case of plastic impact and no load inertia. Knoll and Narud, Ref. d, investigated limit cycles in the phase plane using an analog computer. The investigation of Knoll and Narud covered a wide range of parameters for the case of plastic contact between the gear teeth.

To analytically describe the total system response of a second order servo with backlash, three differential equations and one or more algebraic equations are required. The system as a whole may be treated as being piece wise linear. One differential equation is used to describe the entire system when the gear teeth are in contact and the motor is driving or braking. When the gear teeth are not in contact, two more differential equations are required, one for the motor alone and one for the load alone. The boundary conditions of the differential equations are determined from the solution of one or more algebraic equations, expressing the laws of conservation of momentum and energy. For plastic impact with no bounce of the gear teeth, the law of conservation of momentum must be satisfied. For perfect elastic contact between the gear teeth the law of conservation of momentum and the law of conservation of energy must be satisfied simultaneously. Intermediate cases between perfect elastic and perfect plastic contact can fulfill only the law of conservation of momentum. However, total energy accounting may be made in the intermediate and perfect plastic impact cases.

New, Ref. e, adapted the differential equations of system motion and the law of conservation of momentum (plastic contact) to a digital computer analysis of a second order servomechanism with backlash. The

digital computer was chosen as the method of solution for this investigation primarily because of the flexibility of data presentation and the ease with which cyclic parameter variation could be obtained.

In later sections of this thesis, the differential equations for the system and the algebraic equations for the impact boundary conditions are developed. The computer program and its flow diagram and modes of operation are pointed out. The results of the solutions of problems are presented in several forms and observations are made in Section 5, Results and Discussion. Sample applications of the results to mechanical systems are presented in Section 6, Application of Results.

At this writing, associated work in the area of transient response, primarily peak overshoot and settling time, of a second order servo-mechanism with backlash and plastic and elastic impact is being prepared as a thesis by C. E. Andrews and R. A. Kelley at the U. S. Naval Postgraduate School.

## 2. Development of Equations

The equations developed were those of a phase plane analysis. Prior to defining the net system equations, the assumptions under which the analysis was made will be stated. When necessary these assumptions will be amplified and referred to later in this work.

It was assumed that:

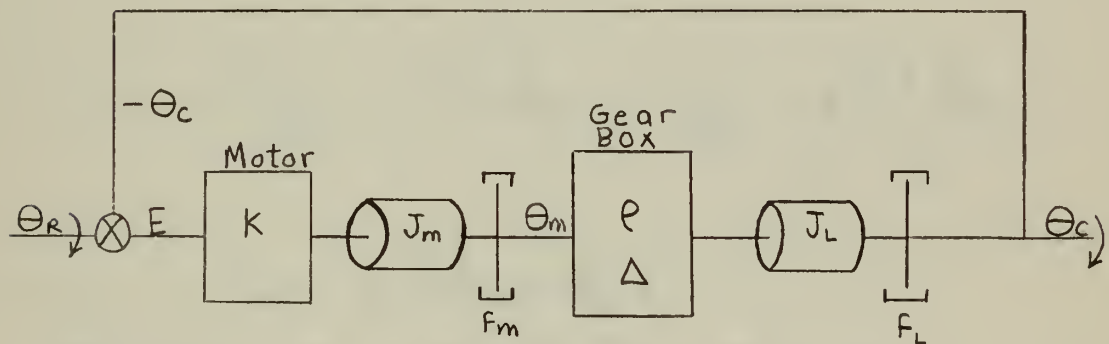
1. The gear teeth were initially in contact and the initial conditions of the system were all equal to zero. This was later proved to be an unnecessary limitation for the study of steady state response.
2. Plastic deformation of the gear teeth during steady contact and impact and any torsional deformations of driving shafts are negligible.
3. The inertias of the gears and drive shafts are considered as part of the load or motor inertia depending on their attachment in the system.
4. The law of conservation of energy was completely satisfied in only the perfect elastic case by maintaining the total mechanical rotational energy of the system constant at the instant prior to and after impact. The law of conservation of momentum is satisfied in plastic, elastic, and intermediate cases. When the law of conservation of momentum is the only equation required to be satisfied, the energy lost from the system is dissipated in the heat of infinitesimal deformations of the gear teeth.
5. The gear teeth are in contact only instantaneously during impact for the elastic and intermediate cases (excluding

plastic) and that the impulse torques of drive, friction and bearing supports, etc., are zero during impact, e.g.

$$\int_{dt \rightarrow 0} T_m dt = 0 \quad \text{and} \quad \Theta = \Theta'$$

6. The coefficient of restitution of the two opposing gear teeth is the same or is described by an equivalent coefficient if the two gear teeth are of unequal coefficients.
7. Backlash is assumed to be equal at all points on the gear circumference. Backlash is measured at the output shaft.

A block diagram of the system considered is presented below.



The equations for the motor and load in contact are:

$$E = \Theta_R - \Theta_C$$

$$T_m = KE = K(\Theta_R - \Theta_C)$$

$$T_L = J_L \ddot{\Theta}_C + f_L \dot{\Theta}_C$$

$$T_{Lm} = \rho T_L = \rho (J_L \ddot{\Theta}_C + f_L \dot{\Theta}_C)$$

$$\Theta_C = \rho \Theta_m \text{ forward drive} \quad \Theta_C = \rho \Theta_m + \Delta \text{ backward drive}$$



$$\dot{\theta}_c = \rho \dot{\theta}_m \quad \text{forward and backward drive}$$

$$\text{hence } T_{Lm} = \rho (J_L \ddot{\theta}_m + f_L \dot{\theta}_m)$$

$$= \rho^2 (J_L \ddot{\theta}_c + f_L \dot{\theta}_c)$$

$$T_m = J_m \ddot{\theta}_m + f_m \dot{\theta}_m + T_{Lm}$$

$$= J_m \frac{\ddot{\theta}_c}{\rho} + f_m \frac{\dot{\theta}_c}{\rho} + \rho^2 (J_L \frac{\ddot{\theta}_c}{\rho} + f_L \frac{\dot{\theta}_c}{\rho})$$

$$= \left( \frac{J_m}{\rho} + \rho J_L \right) \ddot{\theta}_c + \left( \frac{f_m}{\rho} + \rho f_L \right) \dot{\theta}_c$$

$$T_m = K (\theta_R - \theta_c)$$

$$K \theta_R = \left( \frac{J_m}{\rho} + \rho J_L \right) \ddot{\theta}_c + \left( \frac{f_m}{\rho} + \rho f_L \right) \dot{\theta}_c + K \theta_c$$

$$\frac{\ddot{\theta}_c + \left( \frac{f_m}{\rho} + \rho f_L \right) \dot{\theta}_c}{\left( \frac{J_m}{\rho} + \rho J_L \right)} + \frac{K \theta_c}{\left( \frac{J_m}{\rho} + \rho J_L \right)} =$$

$$\frac{K \theta_R}{\left( \frac{J_m}{\rho} + \rho J_L \right)}$$

$$\rho \dot{\theta}_m - \dot{\theta}_c = -\rho \dot{\theta}'_m + \dot{\theta}'_c$$

$$\rho (\dot{\theta}_m + \dot{\theta}'_m) = \dot{\theta}'_c + \dot{\theta}_c$$

For conservation of energy

$$\frac{1}{2} (J_m \dot{\theta}_m^2 + J_L \dot{\theta}_c^2) = \frac{1}{2} (J_m \dot{\theta}'_m{}^2 + J_L \dot{\theta}'_c{}^2)$$

or

$$\frac{J_m}{J_L} (\dot{\theta}_m^2 - \dot{\theta}'_m{}^2) = (\dot{\theta}'_c{}^2 - \dot{\theta}_c^2)$$

and factoring

$$\frac{J_m}{J_L} (\dot{\theta}_m + \dot{\theta}'_m)(\dot{\theta}_m - \dot{\theta}'_m) = (\dot{\theta}'_c + \dot{\theta}_c)(\dot{\theta}'_c - \dot{\theta}_c)$$

The momentum equation which is independent of  $e$  is:

$$\frac{J_m}{\rho J_L} (\dot{\theta}'_m - \dot{\theta}_m) = (\dot{\theta}_c - \dot{\theta}'_c)$$

or rearranging

$$\frac{J_m}{\rho J_L} (\dot{\theta}_m - \dot{\theta}'_m) = (\dot{\theta}'_c - \dot{\theta}_c)$$

The factored energy equation is now divided by the momentum

equation to yield

$$\rho (\dot{\theta}_m + \dot{\theta}'_m) = (\dot{\theta}'_c + \dot{\theta}_c)$$

which checks with the definition of the coefficient of restitution

for  $e=1$ .

The momentum equation implied from the impulse approach that

$$dt \rightarrow 0 \text{ hence } \theta_c = \theta'_c \text{ and } \theta_m = \theta'_m. \text{ The definition of the}$$

coefficient of restitution is independent of position. It may be

noted that the equations for energy and momentum depend only on the

inertia distribution, gear ratio and angular velocities.

In manipulation of these equations it is seen that load inertia may be transferred to the  $\theta_1$ , shaft by multiplication  $\rho^2$ , the same transfer may be accomplished with load friction. Since  $\theta_1$ , and  $\dot{\theta}_1$ , are related to  $\theta_L$  and  $\dot{\theta}_L$  it may be reasoned that the results

for  $\rho = 1$  may be extrapolated to physical systems where  $\rho \neq 1$ .

Since a total energy accounting may be made for the system with the equations used, the rotational energy lost is attributed to the heat of deformation of the gear teeth. This transfer of energy could in fact be determined from the equations used. Thus the conservation of rotational energy  $e=1$  is a special case of a broad interpretation of the law of conservation of energy.

$$\ddot{\theta}_c + 2\zeta\omega_n\dot{\theta}_c + \omega_n^2\theta_c = \omega_n^2\theta_R$$

$$\text{where } \omega_n^2 = \frac{\rho k}{J_m + \rho^2 J_L}$$

$$2\zeta\omega_n = \frac{f_m + \rho^2 f_L}{J_m + \rho^2 J_L}$$

The equation for load alone with gears not in contact is:

$$J_L \ddot{\theta}_c + f_L \dot{\theta}_c = 0$$

$$\text{or } \ddot{\theta}_c + \frac{f_L}{J_L} \dot{\theta}_c = 0$$

The equation for the motor alone with the gears not in contact is.

$$J_m \ddot{\theta}_m + f_m \dot{\theta}_m = k(\theta_R - \theta_c)$$

The method of analysis used by New, Ref. e, is used in order to be able to examine the system independently of its  $\omega_n$

Defining first  $\omega_n t \triangleq t^*$

and differentiating  $\omega_n dt = dt^*$

$$\omega_n = \frac{dt^*}{dt} \quad \text{and} \quad \omega_n^2 = \left(\frac{dt^*}{dt}\right)^2$$

$$\dot{\theta}_c = \frac{d\theta_c}{dt} = \frac{d\theta_c}{dt^*} \frac{dt^*}{dt}, \quad \frac{d\theta_c}{dt^*} = \dot{\theta}_c^*$$

$$\text{hence } \dot{\theta}_c = \dot{\theta}_c^* \omega_n$$



similarly

$$\ddot{\theta}_c = \frac{d^2 \theta_c}{dt^2} = \frac{d^2 \theta_c}{(dt^*)^2} \left( \frac{dt^*}{dt} \right)^2 = \frac{d^2 \theta_c}{(dt^*)^2} \omega_n^2$$

$$\frac{d^2 \theta_c}{(dt^*)^2} = \ddot{\theta}_c^* \quad \text{and} \quad \dot{\theta}_c = \dot{\theta}_c^* \omega_n$$

$$\text{finally } \dot{\theta}_c = \dot{\theta}_c^* \omega_n = \dot{\theta}_c^* \frac{dt^*}{dt}$$

$$\int \dot{\theta}_c dt = \int \dot{\theta}_c^* dt^* \quad \text{or} \quad \theta_c = \theta_c^*$$

Using the equations of the combined system and the load alone,

$$\ddot{\theta}_c + 2\zeta \omega_n \ddot{\theta}_c + \omega_n^2 \theta_c = \omega_n^2 \theta_R \quad \text{combined system}$$

$$\ddot{\theta}_c + \frac{f_L}{J_L} \dot{\theta}_c = 0 \quad \text{load alone}$$

and making the indicated substitution to a transformed (\*) coordinate system for the system equation

$$\omega_n^2 \ddot{\theta}_c^* + 2\zeta \omega_n^2 \dot{\theta}_c^* + \omega_n^2 \theta_c^* = \omega_n^2 \theta_R$$

$$\ddot{\theta}_c^* + 2\zeta \dot{\theta}_c^* + \theta_c^* = \theta_R$$

and introducing the slope equations of the phase plane

$$N_s \dot{\theta}_c^* + 2\zeta \dot{\theta}_c^* + \theta_c^* = \theta_R \quad \text{results.}$$

If the load equation is first put in the form for the phase plane

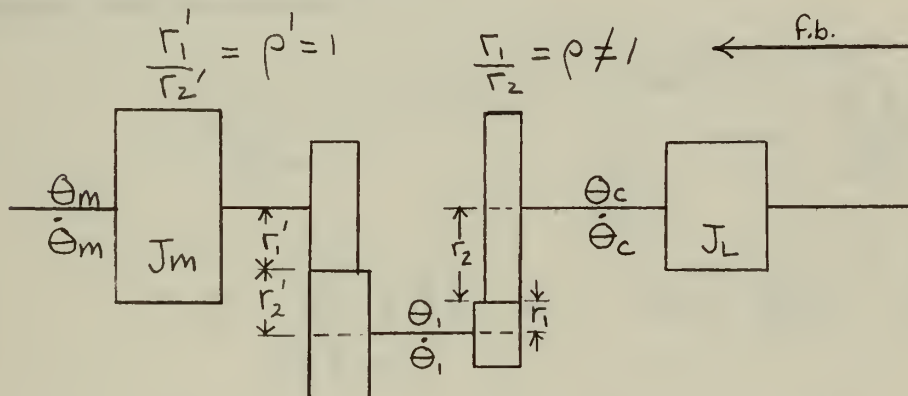
$$N_L \dot{\theta}_c + \frac{f_L}{J_L} \dot{\theta}_c = 0$$

and then transformed to the (\*) coordinate system where  $\dot{\theta}_c^* \neq 0$

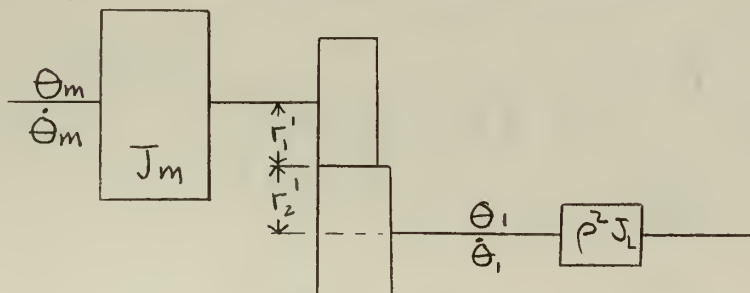
$$N_L \dot{\theta}_c^* + \frac{f_L}{J_L} \dot{\theta}_c^* = 0 \quad \text{results.}$$

The same result may be obtained by setting  $\omega_n = 1$ . The results of such a transformation require inverse scaling for practical application, examples of which given in Section 6. It is noted that the slope of the load-free equation is mathematically the same whether in the phase plane or in the transformed phase plane. It is pointed out at this time that the equations later developed to satisfy the laws of conservation of momentum and energy are independent of the system natural frequency.

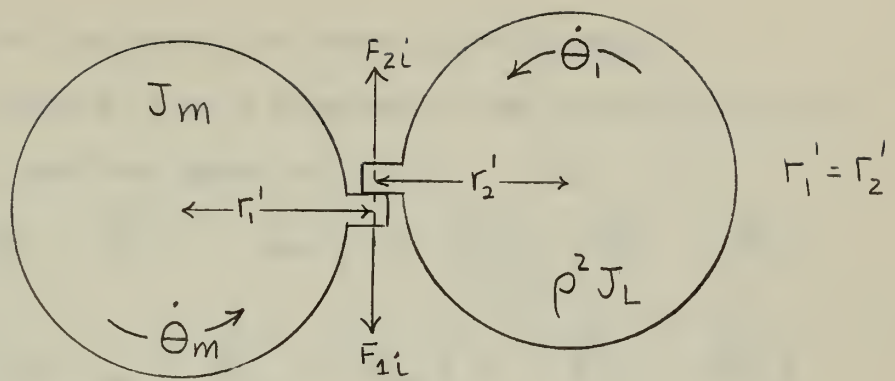
To establish equations for the law of the conservation of momentum and a relationship satisfying the law of the conservation of energy, the following schematic is used.



It is seen that  $\theta_c = -\rho \theta_1$ ,  $\dot{\theta}_c = -\rho \dot{\theta}_1$  and since  $\rho' = 1$  an equivalent schematic results



By representing the inertia of the load and motor as inertia of the gears the following figure is obtained at the instant of impact.



The torque equation may be expressed  $T = J \ddot{\theta} = J \frac{d\dot{\theta}}{dt}$

Impulse is equal to the time rate of change of momentum. The expressions for the rates of change of momentum of the gears treated separately may be written:

$$-\int_{dt \rightarrow 0} |F_{1i}| r_1' dt = \int_{\dot{\theta}_m}^{\dot{\theta}_m'} J_m \frac{d\dot{\theta}_m}{dt} dt$$

$$-r_1' \int_{dt \rightarrow 0} |F_{1i}| dt = J_m (\dot{\theta}_m' - \dot{\theta}_m)$$

$$-\int_{dt \rightarrow 0} |F_{1i}| dt = \frac{J_m}{r_1'} (\dot{\theta}_m' - \dot{\theta}_m) \quad \text{for the motor}$$

and

$$\int_{dt \rightarrow 0} |F_{2i}| r_2' dt = \int_{\dot{\theta}_1}^{\dot{\theta}_1'} \rho^2 J_L \frac{d\dot{\theta}_1}{dt} dt$$

$$\int_{dt \rightarrow 0} |F_{2i}| dt = \rho^2 \frac{J_L}{r_2'} (\dot{\theta}_1' - \dot{\theta}_1)$$

where the primed velocities are those following impact.

If the momentum is to be conserved in the system the impulse functions are equal and opposite.

$$\frac{J_m}{r_1'} (\dot{\theta}_m' - \dot{\theta}_m) = \rho \frac{J_L}{r_2'} (\dot{\theta}_1' - \dot{\theta}_1)$$

and 
$$\frac{J_m}{J_L} \frac{1}{\rho} (\dot{\theta}_m' - \dot{\theta}_m) = (\dot{\theta}_c - \dot{\theta}_c')$$

from substitution of the equations

$$r_1' = r_2', \quad \dot{\theta}_c = -\rho \dot{\theta}_1'$$

The assumption that all other impulse functions are zero at the instant of impact is restated at this point.

A definition of the coefficient of restitution will be made for purposes of this work:

$$e \triangleq - \frac{(\dot{\theta}_m' + \dot{\theta}_1')}{(\dot{\theta}_m + \dot{\theta}_1)} = - \frac{(\rho \dot{\theta}_m' - \dot{\theta}_c')}{(\rho \dot{\theta}_m - \dot{\theta}_c)}$$

It will be shown that this definition of  $e$  will satisfy the law of conservation of energy. Consider the case of  $e=0$ ,

$$0 = -(\rho \dot{\theta}_m' - \dot{\theta}_c'), \quad \rho \dot{\theta}_m' = \dot{\theta}_c'$$

in which case for plastic impact ( $e=0$ ) the gears are moving at the same velocity following impact.

The expression for the coefficient of restitution is examined for the case of perfect elastic contact,  $e=1$ , for which the law of conservation of energy is satisfied.

Solution of the restitution equation for  $e=1$  yields



### 3. Computer Program Development

The physical equations of the net systems were programmed for solution using the Control Data Corporation 1604 high speed digital computer utilizing paper tape program input and magnetic tape output. An IBM 717 line printer was used to extract data from the magnetic tape. The Control Data Corporation machine library and the U. S. Naval Postgraduate School computer subroutine library were used for assembly, Runge-Kutta-Gill numerical integration, and decimal output.

Several changes were made in the forms of the physical equations of the net system in order to eliminate duplicate computing operations and to fit the equations to a form suited to the variable parameters. In the table of symbols, computer mnemonic (m) terms which are used in the assembly subroutine have been indicated by parentheses and will be defined when encountered.

The equation of the system with motor and load combined

$$\ddot{\theta}_c = \omega_n^2 \theta_R - \omega_n^2 \theta_c - 2\zeta \omega_n \dot{\theta}_c$$

was put in the form

$$\ddot{\theta}_c = \omega_n^2 \theta_R - \omega_n^2 \theta_c - \underline{A} \dot{\theta}_c \quad \text{where}$$

$$\underline{A} = 2\zeta \omega_n \quad \text{and}$$

$$\underline{\text{THETAL}} = \theta_c \quad \underline{\text{ONE}} = \theta_R$$

$$\underline{\text{THETALD}} = \underline{U} = \dot{\theta}_c \quad \underline{\text{ZETA}} = \zeta$$

$$\underline{\text{UDOT}} = \ddot{\theta}_c \quad \underline{\text{OMEGANSQ}} = \omega_n^2$$

were the computer m terms used. Underlining will be used in this section to denote the transition to the m terms of the computer equations.

The differential equations were solved using the Runge-Kutta-Gill,

(RUNGE), numerical integration method. The RUNGE subroutine required the definition of synonymous terms for the four iterative cycles used to produce one extrapolated set of variables. The increment of the independent time variable chosen was 0.01 sec. for all  $\delta > 0.1$  and 0.004 sec. for all  $\delta \leq 0.1$ .

The load free equation

$$\ddot{\theta}_c + \frac{f_L}{J_L} \dot{\theta}_c = 0$$

was put in the form

$$\ddot{\theta}_c + \underline{B} \dot{\theta}_c = 0$$

for the computer.

A separate set of RUNGE synonyms was used for the load free equation in order not to destroy the previous computations before they were determined of no further value in computing and also in the making of program decisions.

$\dot{\theta}_c$  was designated V and  $\ddot{\theta}_c$  was designated VDOT for the computer program.

The equation for the motor alone

$$\ddot{\theta}_m + \frac{f_m}{J_m} \dot{\theta}_m + \frac{k}{J_m} \theta_c = \frac{k}{J_m} \theta_R$$

was put in the form

$$\ddot{\theta}_m = - \frac{f_m}{J_m} \dot{\theta}_m + \frac{k}{J_m} (\theta_R - \theta_c)$$

$$\ddot{\theta}_m = - \underline{D} \dot{\theta}_m + \underline{C} (\theta_R - \theta_c)$$

and for RUNGE, the terms

$$\theta_m = \underline{\text{THETAM}}$$

$$\dot{\theta}_m = \underline{\text{THETADM}} = \underline{W}$$

and

$$\ddot{\theta}_m = \underline{\text{WDOT}}$$

were used.

The equation representing the law of conservation of momentum and the definition of coefficient of restitution were combined to the forms

$$\dot{\theta}'_c = \frac{J_m}{J_m + \rho^2 J_L} \left[ \rho \dot{\theta}_m (1+e) + \dot{\theta}_c \left( \rho^2 \frac{J_L}{J_m} e \right) \right]$$

$$\dot{\theta}'_m = \frac{\dot{\theta}'_c - e \rho \dot{\theta}_m + e \dot{\theta}_c}{\rho}$$

for the computer program. No provision for additional m terms was made to denote the primed values. The additional terms:

$$e = \text{RESTITUT} \quad \rho = \text{RHO} \quad \text{and} \quad \rho^2 = \text{RHOSQ}$$

were used.

Two equations were used to define the boundaries of operation of the load and motor when they were acting separately,

$$\theta_c = \rho \theta_m \quad \text{and} \quad \theta_c = \rho \theta_m + \Delta$$

Two major decisions of the computing cycle were the determination of the point where the motor and load would float free, FLOATEST, and the response of the system to impact of the load and motor gears when the boundary conditions on position were met, COMBTST.

The first decision, the point of float free, is made on the equality or inequality of slopes in the phase plane, i.e., at separation  $N_L = N_S$ . Equating the slopes:

$$\frac{\omega_n^2}{\dot{\theta}_c} (\theta_R - \theta_c) - \underline{A} + \underline{B} = 0$$

The above equation is solved after computing each point in the combined phase trajectory. If  $N_S + N_L > 0$  the system remains combined. If  $N_S + N_L \leq 0$  the system separates, having the response motor alone and load alone immediately thereafter.

The second major decision, that of whether after impact the

response should be that of the combined system or the response of the load and motor acting separately, is made on the basis of the resultant velocities after impact and upon which side of the backlash the motor and load positions are found.

The computer program, its flow diagram, a description of the program operation, and an explanation of the readout of information with examples are presented in the Appendix.



#### 4. Methods and Scope of Examination

Two modes of operation of the computer program were used. The first mode considered was the phase trajectory. This mode was used to check the operation of the computing cycle. Time, velocity and position of the load were printed when the system was combined. When the system was separated, the values of motor velocity and motor position were also printed. Although the time increment used for computations was either 0.01 sec. or 0.004 sec., printed outputs for the first mode were taken only every 0.1 sec. Additional printed outputs were taken at the time of contact of the gears just prior to and immediately following the solution of the momentum and restitution equations. A typical printout of the phase trajectory mode of operation is shown in Appendix C.

The second mode of operation was used for printing only the maximum computed positive load overshoot position for each cycle, the associated load velocity and the exact problem time of the computation. A sufficient number of print-outs for each problem was obtained to prove limit cycle existence and average size. Since this investigation was concerned primarily with the steady state response, the second mode of operation was the one utilized to obtain the majority of the data. This read-out method markedly decreased the data reduction time for the problem analysis, as opposed to the analysis of steady state conditions provided by a full phase trajectory print-out. Since the time required for computer read-out was several magnitudes greater than the computing time, valuable computer time was saved by this mode of operation. An example of the printed output obtained when only maximum load position overshoot was of interest is given in Appendix D. Approximately 1400

phase trajectories were solved using this mode of operation. Each solution required an average of three minutes of computer time.

For the purpose of general examination of the problem it was assumed that  $\rho = 1$ ,  $K = 1$ ,  $\omega_n = 1$ ,  $J_m + \rho^2 J_L = 1$ . Specific solutions were made with  $\rho \neq 1$ ,  $\omega_n \neq 1$  to determine scaling effects. The results of these solutions are presented in Section 6, Application of Results.

The program can also be made to solve linear systems by setting  $\Delta = 0$ .

The major variable parameters used in this investigation were:

$$\zeta = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0$$

$$\frac{\rho^2 J_L}{F_T} = 0, 0.2, 0.4, 0.6, 0.8, 1.0$$

$$\frac{\rho^2 J_L}{J_m} = \frac{0.1}{0.9}, \frac{0.2}{0.8}, \frac{0.5}{0.5}, \frac{0.8}{0.2}, \frac{0.9}{0.1}$$

$$\Delta = 0.3 \text{ radians}$$

$$e = 0, 0.6, 0.8, 1.0$$

Additional parameters of

$$\frac{\rho^2 J_L}{F_T} = 0.04, 0.1, 0.9; \text{ and } \Delta = 0.01, 0.03, 0.01, 0.15 \text{ radians}$$

were used in certain instances to examine particular characteristics of the response.

The program was limited somewhat in that the value of  $\frac{\rho^2 J_L}{J_m} = \infty$  and zero were excluded due to generation of undefined mathematical quantities.

The value of backlash was made abnormally large,  $\Delta = 0.3$  radians, to allow an easier interpretation of the non-linear response. Since the influence of  $\Delta$  was linear, this caused no inaccuracies. All

graphs for limit cycle size were plotted with  $\Delta = 0.3$  radians. The effect of values for  $\Delta$  other than 0.3 is also shown in graphical form.

## 5. Results and Discussion

Two types of system response were observed. The first type was a convergence to zero error, which was a case of static and dynamic equilibrium of the system. This is a characteristic of a linear second order servomechanism. The second type of system response observed was the divergence or convergence to a state of dynamic equilibrium. This second type of response is termed a limit cycle and is characterized by cyclic travel of the load through the same points of the phase trajectory. When the second mode of computer operation was used, both the maximum load position per cycle and the period of the recorded position were analyzed to determine if the characteristic response was a limit cycle. Exact repetition of cyclic values was impossible due to the numerical methods used and the computer round-off error for the output routine. Typical examples of the limit cycle and no limit cycle response are given in Appendices D 1 and D 2.

No case of dynamic instability or divergence of the system without bound was observed in the investigation. Intuition might lead to this same conclusion when the following unique characteristics of this system are pointed out: (a) As a maximum limit, energy was conserved at the instant of impact. (b) Energy was supplied to the system only in finite amounts and at a finite rate by the error detector. (c) Energy sinks were present in the system frictional elements while energy storage units were present in both the motor and load. (d) No undefined limits appeared in the applied physical equations,

$$\rho^2 \frac{J_L}{J_m} = 0 \text{ and } \infty \text{ excluded.}$$



The results of the investigation are offered in the form of charts for parameter areas of limit cycle existence and nonexistence in Fig. 1 for  $e = 0., 0.6, 0.8$  and Fig. 2 for  $e = 1.0$ . The parameters  $\rho^2 \frac{f_L}{F_T}$  and  $\rho^2 \frac{J_L}{J_m}$  are designated abscissa and ordinate respectively. The parameter points of examination for which solutions were obtained are circled. The zone enclosed by a particular  $\zeta$  line is largely an interpretation by the authors of the results of the investigation, and shows the area in which no limit cycle existed. Due to the large change of the variable parameter values between solutions, the charts are not exact. All circled intersections interior to the  $\zeta$  line indicate parameters which resulted in no limit cycles, e.g., Fig. 1 for  $e = 0, 0.6, 0.8$  at a  $\zeta = 0.4$ , the values of  $\rho^2 \frac{f_L}{F_T} = 0.6, 0.8$  at a  $\rho^2 \frac{J_L}{J_m} = 9.0$  and the value of  $\rho^2 \frac{f_L}{F_T} = 0.6$  at a  $\rho^2 \frac{J_L}{J_m} = 4.0$  resulted in no limit cycle. Similarly, for  $\zeta = 0.5$ , the values of  $\rho^2 \frac{f_L}{F_T} = 0.4, 0.6$ , and a  $\rho^2 \frac{J_L}{J_m} = 1.0$  as well as all other parameter intersections interior to the  $\zeta = 0.5$  curves, resulted in no limit cycles.

All circled parameter intersections exterior to particular  $\zeta$  lines define parameters which were examined and resulted in a steady state limit cycle. The area to the right of the single  $\zeta = 1.0$  line designates the parameters for which no limit cycles were observed. Limit cycles were observed for all  $\zeta \leq 0.3$  for the parameters examined.

The investigation was made with  $\Delta = 0.3$  radians; however, results of the investigation of variation of limit cycle size indicate

that existence of the limit cycle is independent of the size of backlash in a nonlinear system for a  $\Delta$  greater than zero. Variation of  $\Delta$  had the effect of a linear variation of limit cycle size only.

The zones depicted on the existence charts are approximately represented by straight lines; however there is reason to believe that a complete examination would show a curvature in the lines. This observation will be elaborated upon later when the relationship between the existence charts and the figures of limit cycle size is discussed.

From Fig. 1 it is seen that there is only a very small difference in the limit cycle existence zones for plastic impact ( $e = 0$ ) and the intermediate values of the coefficient of restitution,  $e = 0.6$  and  $0.8$ . When comparing the elastic impact ( $e = 1$ ) in Fig. 2 to the plastic and intermediate cases in Fig. 1, the most obvious point of comparison is the drastic reduction of the areas in which limit cycles did not result for  $e = 1.0$ . However, it may be generally observed that with any  $e$  held constant, the area in which a limit cycle could not be obtained increased with the increase of system  $\int$ .

For all values of  $e$ , there is an apparent common enclosed zone centroid which is described by the system parameters  $\rho^2 f_L / F_T = 0.5$  and  $\rho^2 J_L / J_m = 1.0$ . These parameters result in the system time constants  $f_L / J_L = f_m / J_m$ , since  $\omega_n = 1$  and  $\rho = 1$  were chosen for the investigation. There is also an apparent symmetry about the centroid of opposite quadrants which can be related to the fact that since  $\omega_n = 1$  and  $\rho = 1$ , the points diametrically opposite across the centroid of the figure (using the decimal values of the parameter intersection) represent an exact exchange of component time constants. (See Fig. 2 for example). Two diametrically opposite points may have

exactly the same motor and load time constants; however, they are descriptive of two different systems since the friction and inertia distributions are different for the two points.

Figs. 3 through 26 present the maximum positive error in radians of the limit cycle for a unit step input and with backlash equal to 0.3 radians. The use of the charts is explained in Section 6, Application of Results. Each figure is presented with  $\rho_{f_L/F_T}^2$  and  $e$  constant. The independent parameter  $\rho_{J_L/J_m}^2$  is indicated as abscissa and curves of constant  $S$  are plotted with limit cycle error as the ordinate. The results were presented on a log log plot since the observed limit cycle errors encompassed a range from 0.01 to 1.0 radians, and the appearance and disappearance of limit cycles and the change of limit cycle error was more readily apparent on the log log form.

In general as  $e$  increases, the size of limit cycle decreases, all other parameters remaining constant. However this is not without a few exceptions; the generalization is most accurate for large size limit cycles. It may be noted that when viewing the individual charts, if a limit cycle exists at very low  $\rho_{J_L/J_m}^2$  ratio there is a tendency for the same system to have a smaller limit cycle for higher  $\rho_{J_L/J_m}^2$  ratio. In addition, for these same conditions, if limit cycles exist for several values of  $S$ , there is a convergence toward the same size of limit cycles for each  $S$  as  $\rho_{J_L/J_m}^2$  increases. At the value  $\rho_{f_L/F_T}^2 = 0$ , a limit cycle will always occur for any value of  $e$  or  $S$ .

Viewing the charts in sequence, with either  $e$  constant or  $\rho_{f_L/F_T}^2$  constant, it is noted that there is continuity of pattern flow between



adjacent charts. This continuity between charts appears to form a symmetrical pattern of constant  $\zeta$  lines which can be interpolated for intermediate values. When the figures for  $\rho^2 f_L/F_T = 0.4$  and  $0.6$  are analyzed, this symmetrical pattern is observed in each chart about  $\rho^2 J_L/J_m = 1$ . The limit cycle decreases and disappears at the opposite extremes of  $\rho^2 J_L/J_m$ . The charts of  $\rho^2 f_L/F_T = 0.4$  and  $0.6$  are symmetrical to each other, while  $\rho^2 f_L/F_T = 0.2$  and  $0.8$  are similar. At the extreme values of  $\rho^2 f_L/F_T = 0$  and  $1.0$ , no symmetrical comparison can be made. An additional point of symmetry about  $\rho^2 f_L/F_T = 0.5$  can be visualized for all values of  $e$ . These parameters for symmetry are noted in the discussion of the existence charts. The irregularities in the curves of medium values of  $\zeta$  can not be explained by the authors; however, it may be mentioned again that the points of symmetry are characterized by exact exchange of component time constants.

The noted symmetry of the limit cycle size charts shows a direct relationship to the symmetry found in the existence charts. Since the existence charts show contours of constant  $\zeta$  for a surface of zero limit cycle size, one is led to the possibility of a three dimensional presentation to show a variation of limit cycle size for the same parameters used in the existence charts. Such a three dimensional figure for constant  $e$  using  $\rho^2 f_L/F_T$ ,  $\rho^2 J_L/J_m$  and  $\zeta$  as the axes is shown in Fig. 27. Other surfaces could be added above this zero limit cycle "floor" to show the variation of a given limit cycle size by contours of  $\zeta$ . The ultimate result of this line of investigation would be the evaluation of the poles and zeros for a conformal mapping presentation. Unfortunately, time did not permit the authors to investigate



this avenue further.

The results of the investigation of the dependence of limit cycle size on the amount of backlash are presented in Figs. 28 and 29 using coefficients of restitution of 0 and 1.0 at a  $\xi = 0.6$ . A thorough investigation was made at this  $\xi$  since it is an average system parameter often used in design work. The limit cycle size was found to vary directly with backlash size without exception between the values of 0.3 and 0.01 radians. When limit cycles existed at  $\Delta = 0.3$ , there was no disappearance of limit cycle with decreasing magnitude of  $\Delta$ . Where no limit cycle existed, at  $\Delta = 0.3$ , limit cycles continued to be non-existent throughout the range of  $\Delta$  examined. Several solutions of problems with  $\xi = 0.6$  and  $e = 0.6 \& 0.8$  and  $\xi = 0.1 \& e = 1.0 \& 0$  with various  $\Delta$  values, proved the linear variation of limit cycle size with backlash size for all values of  $e$ .

When a servomechanism under steady state conditions is disturbed by a small perturbation, it will return to the original steady state conditions. However, the transient response due to the perturbation may be quite different from the transient response which occurred due to the original signal input. The concepts of energy and static and dynamic equilibrium mentioned in Section 1 may be better understood as a result of the perturbation technique if the usual initial conditions of  $\dot{\theta}_c(0) = \theta_c(0) = 0$  are considered as a disturbance from the stability conditions predetermined by selection of system parameters and signal input.

This same small perturbation technique was simulated mathematically using the digital computer to obtain several solutions. The cases examined and their initial conditions were:

$$\begin{aligned}
 \text{a) } \zeta &= 0.6, \quad \rho_{J_L/J_m}^2 = 0.8/0.2, \quad \rho_{f_L/F_T}^2 = 0, \quad e = 0, \quad \Delta = 0.3 \\
 \dot{\theta}_c(0) &= 0, \quad \theta_c(0) = 1.2 \\
 \dot{\theta}_m(0) &= 0, \quad \theta_m(0) = 1.0
 \end{aligned}$$

system separated.

$$\begin{aligned}
 \text{b) } \zeta &= 0.6, \quad \rho_{J_L/J_m}^2 = 0.1/0.9, \quad \rho_{f_L/F_T}^2 = 1.0, \quad e = 1.0, \quad \Delta = 0.3 \\
 \dot{\theta}_c(0) &= 1.0, \quad \theta_c(0) = 1.0 \\
 \dot{\theta}_m(0) &= 1.0, \quad \theta_m(0) = 1.0
 \end{aligned}$$

system combined

$$\begin{aligned}
 \text{c) } \zeta &= 0.4, \quad \rho_{J_L/J_m}^2 = 0.5/0.5, \quad \rho_{f_L/F_T}^2 = 0.4, \quad e = 0.6, \quad \Delta = 0.3 \\
 \dot{\theta}_c(0) &= 0, \quad \theta_c(0) = 1.005 \\
 \dot{\theta}_m(0) &= 0, \quad \theta_m = 1.00
 \end{aligned}$$

system separated

In the three cases examined, the same limit cycle was obtained, both in size and period, as that resulting from the normally used initial conditions,  $\dot{\theta}_c(0) = \theta_c(0) = 0$ . The maximum difference between the perturbation response and the original limit cycle size for the three cases was  $18 \times 10^{-5}$  radians. Two types of transient response were observed. These were convergence to limit cycle in the first two cases and divergence to limit cycle in the last case.

The results obtained by the small perturbation technique supported the validity of the analysis of the problem by digital methods. This result also concurs with the findings for the case of  $e = 0$  by Knoll and Narud, Ref. d, in that the steady state limit cycle size is completely specified by the choice of system parameters and independent of system initial conditions.

Mention is made here that the computer program could not be run normally for the perturbation method. The computer was stopped after

the print out of system constants, then the initial conditions of the servo system were inserted manually. The program was then restarted at the desired point for computing, either as a combined system or with the motor and load separated. The program was then allowed to operate to the normal completion of the solution.

The work of Ref. d reported that the system transient response for the case  $e = 0$  arrived at a steady state limit cycle by two possible phase trajectories. One of these phase trajectories was smooth convergence to the limit cycle, while the second was a converging overshoot toward the inside of the limit cycle followed by a divergence to the limit cycle. From the results of this investigation for the cases  $e = 0, 0.6, 0.8$  and  $1.0$ , similar transient indications were obtained.

Several computer solutions were accomplished for the cases  $\Delta = 0.3$  radians,  $e = 0, 1.0$ ,  $\rho = 1.0$ ,  $\omega_n = 1.0$ ,  $K = J \neq 1.0$ . In addition various combinations of  $\rho \neq 1.0$  and  $\omega_n \neq 1.0$  were obtained. The purpose of these solutions were:

1. To determine the general applicability of the computer program.
2. To verify the scaling developed in Section 2, Development of Equations.
3. To determine the applicability of the results of this thesis.

The following results were determined:

A. Two solutions were obtained ( $e = 0$  &  $1.0$ ) for the case  $\xi = 0.4$ ,

$$\rho = 1.0 \quad \omega_n = 1.0, \quad J = 2.0, \quad K = 2.0, \quad \beta^2 \frac{f_L}{F_T} = 0.4, \quad \rho^2 \frac{J_L}{J_m} = 4.0.$$

The results of these solutions were the same (time, load velocity, load

position, and limit cycle size) as the solutions obtained for  $\beta^2 \frac{f_L}{F_T} = 0.4$ ,

$$\rho^2 \frac{J_L}{J_m} = 4.0, \quad \xi = 0.4 \text{ using the nondimensional parameters } \omega_n = 1.0, \\ \rho = 1.0, \quad J = 1.0, \quad K = 1.0.$$



B. Solutions were obtained for  $e = 0$  &  $1.0$  for the case  $\mathcal{S} = 0.4$ ,

$$\rho = 1.0, \omega_n = 2.0, J = 1.0, K = 4.0, \rho^2 \frac{f_L}{F_T} = 0.4, \rho^2 \frac{J_L}{J_m} = 4.0.$$

The results of these solutions yielded the same size maximum overshoots and limit cycle size as the nondimensional solutions for  $\mathcal{S} = 0.4$ ,

$$\rho^2 \frac{f_L}{F_T} = 0.4, \rho^2 \frac{J_L}{J_m} = 4.0.$$

The times of maximum overshoots in this case were one-half the times of occurrence of the same overshoots for the nondimensional solution. The velocity at a maximum overshoot was twice the value of velocity obtained for the same overshoot in the nondimensional solution. This same comparison can be made for Cases A and B. The results of these and other solutions prove the validity of formulae used in Section 2, Development of Equations, concerning the transformation to the coordinate system. (\*)

C. Solutions were obtained for the case  $e = 0$  &  $1.0$ ,  $\mathcal{S} = 0.4$ ,  $\rho = 0.5$ ,

$$\omega_n = 1.0, \frac{f_L}{F_T} = 0.8, \frac{J_L}{J_m} = 4.0, \rho^2 \frac{f_L}{F_T} = 0.2, \rho^2 \frac{J_L}{J_m} = 1.0.$$

The results of these solutions were the same (time, load velocity, maximum overshoots and limit cycle size) as for the nondimensional solutions

$$\mathcal{S} = 0.4, \rho = 1.0, \omega_n = 1.0, \rho^2 \frac{f_L}{F_T} = 0.2, \rho^2 \frac{J_L}{J_m} = 1.0.$$

D. Solutions were obtained for the case  $e = 0$  &  $1.0$ ,  $\mathcal{S} = 0.4$ ,

$$\rho = 0.5, \omega_n = 2.0, \frac{f_L}{F_T} = 0.8, \frac{J_L}{J_m} = 4.0, \rho^2 \frac{f_L}{F_T} = 0.2, \rho^2 \frac{J_L}{J_m} = 1.0.$$

The results of these solutions yielded the same size maximum overshoots and limit cycle size as the nondimensional solutions for  $\mathcal{S} = 0.4$ ,

$\rho^2 \frac{f_L}{F_T} = 0.2, \rho^2 \frac{J_L}{J_m} = 1.0.$  The times and velocities in this case were scaled by  $\omega_n$  as in Case B.

Several phase trajectories using  $e = 0$  were compared with those obtained by New, Ref. e. Generally good correlation was obtained on the value of limit cycle error and in the transient response. Small disagreements were expected, resulting primarily from differences in the

programming procedures and analysis of results by read-out methods.

Sample phase trajectories for the cases of  $e = 0$  &  $1.0$  are presented in Figs. 30 and 31. Although the transient analysis of the system is beyond the scope of this work, these figures may aid in understanding the physical problem.

Of academic interest was the accidental operation of the computer program for several solutions using  $\xi = 0$ . Several responses were observed. One was a nearly pure oscillatory system from the first overshoot and thereafter another was a very slow convergence or slow divergence of the oscillations after the first overshoot. The convergence or divergence probably resulted from errors generated in numerical iterations and approximations.



## 6. Application of Results

In this section, the results of this thesis will be applied to hypothetical physical systems. Methods will be indicated by which physical systems may be analyzed or designed. Attention is called to the assumptions stated in Section 2 under which this study was made.

It is desirable to find an equation relating certain system variables, therefore the equations:

$$(1) \quad \omega_n^2 = \frac{\rho k}{J_m + \rho^2 J_L} = \frac{\rho k}{J_m} \left[ \frac{1}{1 + \rho^2 \frac{J_L}{J_m}} \right]$$

and

$$(2) \quad 2 \zeta \omega_n = \frac{f_m + \rho^2 f_L}{J_m + \rho^2 J_L} = \frac{\rho^2 f_L \left[ \frac{f_m}{\rho^2 f_L} + 1 \right]}{J_m \left[ 1 + \rho^2 \frac{J_L}{J_m} \right]}$$

will be manipulated to the form

$$(3) \quad \zeta = \frac{1}{2} \frac{\rho^2 f_L}{J_m} \frac{\left[ \frac{f_m}{\rho^2 f_L} + 1 \right]}{\left[ 1 + \rho^2 \frac{J_L}{J_m} \right]} \frac{\sqrt{J_m}}{\sqrt{\rho} \sqrt{k}} \sqrt{1 + \rho^2 \frac{J_L}{J_m}}$$

or

$$(4) \quad \zeta = \frac{1}{2} f_L \left[ \frac{f_m}{\rho^2 f_L} + 1 \right] \left[ \frac{\rho^3}{J_m k \left( 1 + \rho^2 \frac{J_L}{J_m} \right)} \right]^{1/2}$$

In view of the form of Equation (4) for  $\zeta$  and recalling that the variable parameters used on the figures were  $\rho^2 \frac{J_L}{J_m}$  and  $\rho^2 \frac{f_L}{F_T} =$

$\frac{\rho^2 f_L}{f_m + \rho^2 f_L} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1}$  it is seen that the expression for  $\zeta$  in fact includes the other two variable parameters.

From discussion concerning the effects system natural frequency, it will be recalled that the value of overshoot and hence limit cycle are not affected by  $\omega_n$ ; however, the time and velocity are scaled for  $\omega_n \neq 1.0$ . Thus the transient response characteristics may be related to the same parameters used in these applications.

If a motor and load are selected for a system and the limitation is imposed that system operation shall not result in a limit cycle, Equation (4) and the limit cycle existence charts may be used to determine the unspecified system parameters  $\zeta$ ,  $K$  and  $\rho$ . The coefficient of restitution for the proposed gear train material may be determined by the use of the equation in Section 2.

To proceed, select a value of  $K$  and  $\rho$ , then determine the value of  $\zeta$  from Equation (4). Enter the limit cycle existence chart for the value of  $e$ , system  $\zeta$  and the parameters  $\rho^2 \frac{J_L}{J_m}$  and  $\frac{1}{\frac{\zeta_m}{\rho^2 f_L} + 1}$ . Determine if the selected parameters describe a point of no limit cycle. Since limit cycle existence is independent of backlash ( $\Delta \neq 0$  excluded), backlash is not a variable parameter. Successive trial values of the unspecified parameters may be required to meet the limitation of no limit cycle. If it is also required that certain transient response characteristics be met, these conditions could be examined by a similar method.

In the event that no acceptable parameters are found to satisfy the system requirements for no limit cycle, the figures for limit cycle size may be examined to obtain a minimum size limit cycle. The method of solution is similar to that previously described.

Example: The material intended for use in the gears has an  $e = 0.6$ .

The motor and load have the parameters  $f_m = 0.64$ ,  $J_m = 0.25$ ,  $f_L = 0.64$ ,  $J_L = 1.0$  and it desired that the system operate with no limit cycle.

If the values are selected  $K = 4.0$ ,  $\rho = 0.5$ , the values  $\rho^2 \frac{J_L}{J_m} = 1.0$  and  $\frac{f_m}{\rho^2 f_L} = 4.0$ ,  $\frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$  result.

Solve Equation (4) for  $\xi = .5(.64)(5+1) \frac{.5^{3/2}}{[.25(4)(1+1)]^{1/2}}$ , thus

$\xi = 0.4$ . Enter the chart for limit cycle existence, Fig. 1,

$e = 0.6$ ,  $\xi = 0.4$ ,  $\frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$ ,  $\rho^2 \frac{J_L}{J_m} = 1.0$

It can be seen that limit cycle will exist for these parameters.

Since  $K$  is most easily varied in equation (4), a new  $K$  is selected at 1.0 in an effort to find parameters for no limit cycle. It is seen that

$$\xi_{K=1.0} = \xi_{K=4.0} \sqrt{\frac{4}{1}} = 0.4(2)$$

with all other parameters constant, thus  $\xi = 0.8$  Enter Fig. 1

with  $e = 0.6$ ,  $\xi = 0.8$ ,  $\rho^2 \frac{J_L}{J_m} = 1.0$ ,  $\frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$

These parameters describe system operation with no limit cycle and are well within the area limits. The transient response of this system may be undesirable and various combinations of  $\rho$  and  $K$  would have to be tested to satisfy the additional requirements of the problem. It will be restated that the existence areas of Figs. 1 and 2 are not exact.

The figures of limit cycle size may be used in the same manner as the existence charts. The same parameters of the first case are

assumed:  $K = 4.0$ ,  $\rho = 0.5$ ,  $\rho^2 \frac{J_L}{J_m} = 1.0$ ,  $\frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$ ,  $\xi = 0.4$

$e = 0.6$ ,  $\Delta = 0.1$  radians. Enter Fig. 10 with the parameters given.

The ordinate, Limit Cycle Error  $\left( \frac{0.3}{\Delta} \right) = 0.13$  radians, is obtained.

The maximum positive error in the limit cycle  $= 0.13 \frac{(0.1)}{(0.3)} = 0.0434$

radians. Enter the same figure for  $e = 0.6$ ,  $\zeta = 0.8$ ,  $\rho^2 \frac{J_L}{J_m} = 1.0$ ,

$\frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$ ,  $\Delta = 0.5$  radians; this is the second case of exami-

nation for existence. No  $\zeta = 0.8$  curve exists; hence no limit cycle

exists for this choice of parameters for any nominal value of backlash.



## 7. Conclusions

Within the limitations of the assumptions made and the scope of this thesis, it is concluded that:

- A. Two types of system steady state response result. The type of response is independent of backlash size, system natural frequency and initial conditions. The types of response are convergence to signaled input position and convergence to a limit cycle.
- B. If a limit cycle exists in the steady state response of a system, the size of the limit cycle varies directly with the size of backlash, all other parameters constant.
- C. The greatest change in system steady state response occurred for the case when system rotational energy is conserved ( $e = 1.0$ ).
- D. A point of symmetry for the type of system steady state response exists with equal motor and load time constants. For medium to high values of system damping coefficient, symmetry with respect to limit cycle size exists for an exchange of motor and load time constants.
- E. The results of this thesis are presented in a form which may aid in system design or analysis.



## 8. Recommendations

From the results of this thesis it can be seen that there remain many fertile areas for further investigation. For studies of this type it is strongly recommended that the digital computer be used because of its speed and versatility. The limits of investigation depend to a great extent on the ingenuity of the programmer. Some recommended areas of future investigation are:

a) More precise definition of the limit cycle existence zones and a more thorough coverage of the limit cycle size of similar systems having the same type of parameters. A three-dimensional presentation as mentioned in the discussion could be obtained.

b) Include torsion and other deformations of the mechanical system in the physical equations of the system to determine their affect on the system response.

c) An analysis of the limit cycle existence zone and transient response of a system to determine if there is a relationship between the two. An analytical expression for the possibility of limit cycle existence might be developed in terms of system parameters.

d) Examine the transient and frequency response of the system due to various  $\Theta_R$  input functions. This would require minor modifications to the computation section of the existing computer program.

e) Since the physically realizable quantities of position, velocity and acceleration of both the motor and load are available as computed quantities, the study of the compensation problem and its optimization are obvious areas for further consideration.

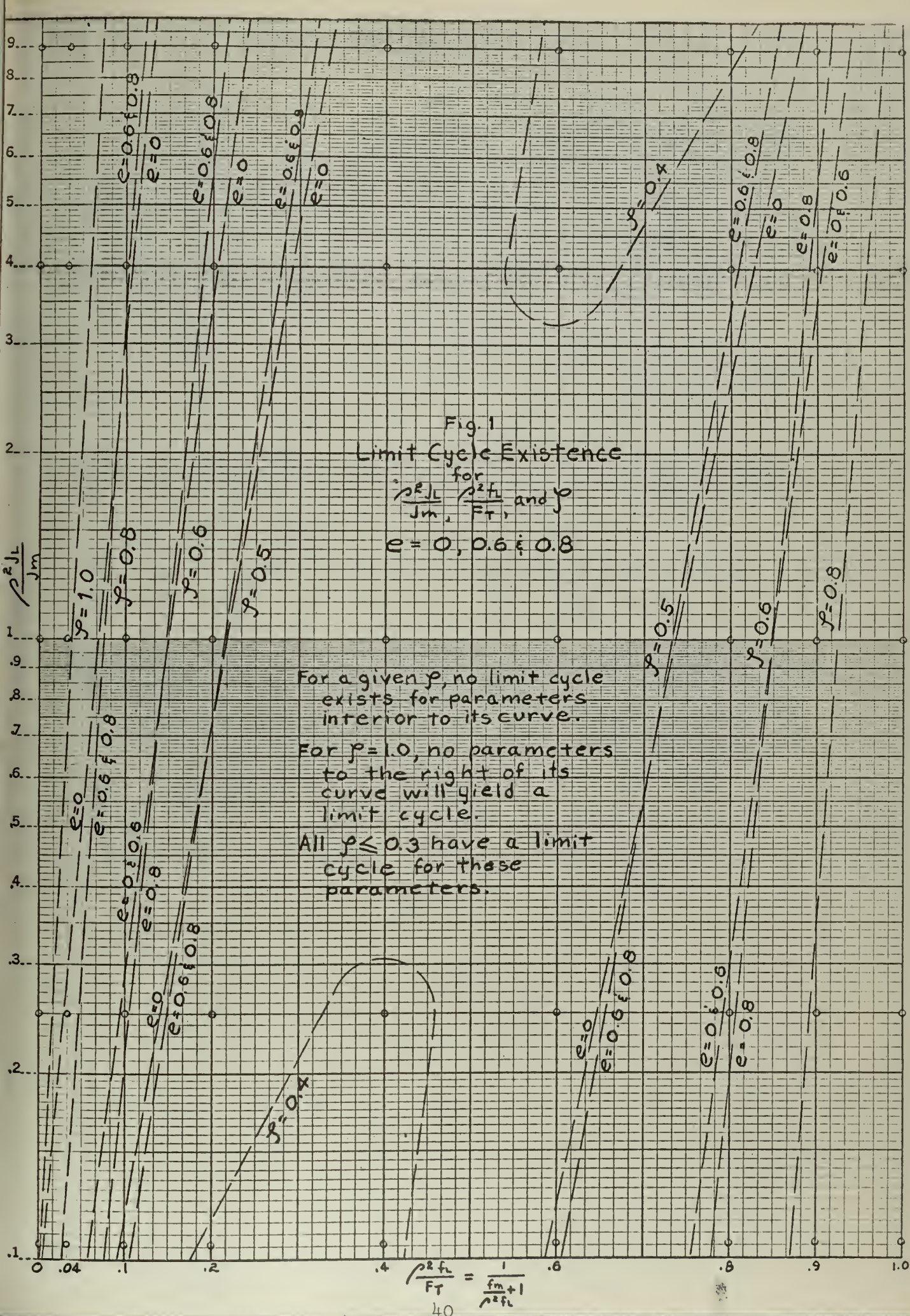
f) The general study of other higher order nonlinear systems is suitable for digital computer programming since most numerical inte-

gration methods will handle systems having equations of any order.

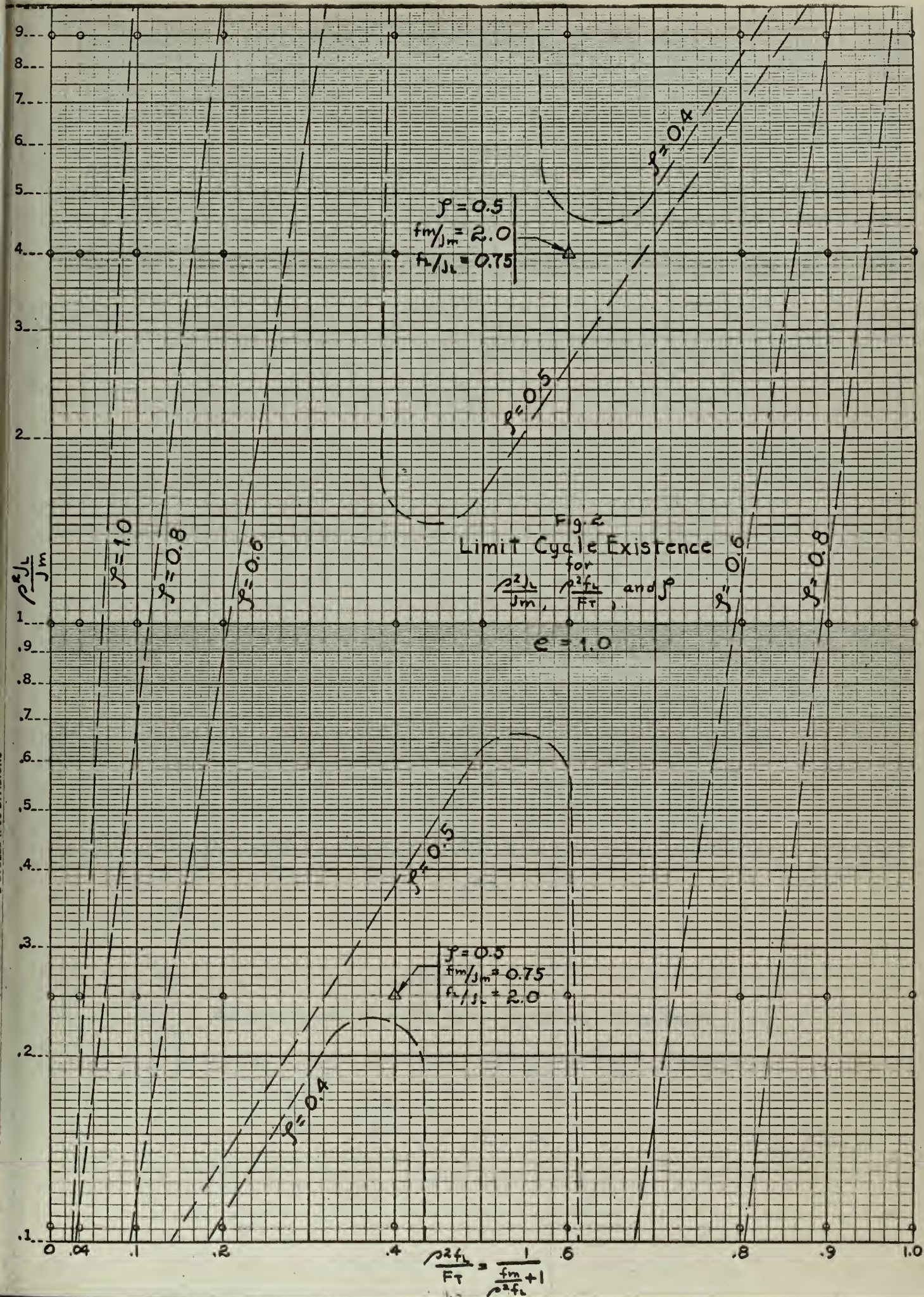
## 9. REFERENCES

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- b. Lutkenhouse, W.J., "Dividing Lines for Backlash in the Phaseplane", Unpublished Master's Thesis, United States Naval Postgraduate School, 1959.
- c. Pastel, M.P. and Thaler, G.J., "Instrument Servomechanisms with Backlash, Couloumb Friction, and Stiction", Trans. A.I.E.E. (Applications and Industry) July 1960.
- d. Knoll, A.L. and Narud, J.A., "Phase Plane Investigation of a Servomechanism with Backlash between Motor and Load", Technical Report 309 for Office of Naval Research, NR-375-017, Cruft Laboratory, Harvard University, Cambridge, Massachusetts, July 30, 1959.
- e. New, N.C., "Effects of Backlash in the Second Order Servo", Unpublished Master's Thesis, United States Naval Postgraduate School, 1960.











Limit Cycle Error (Radians)

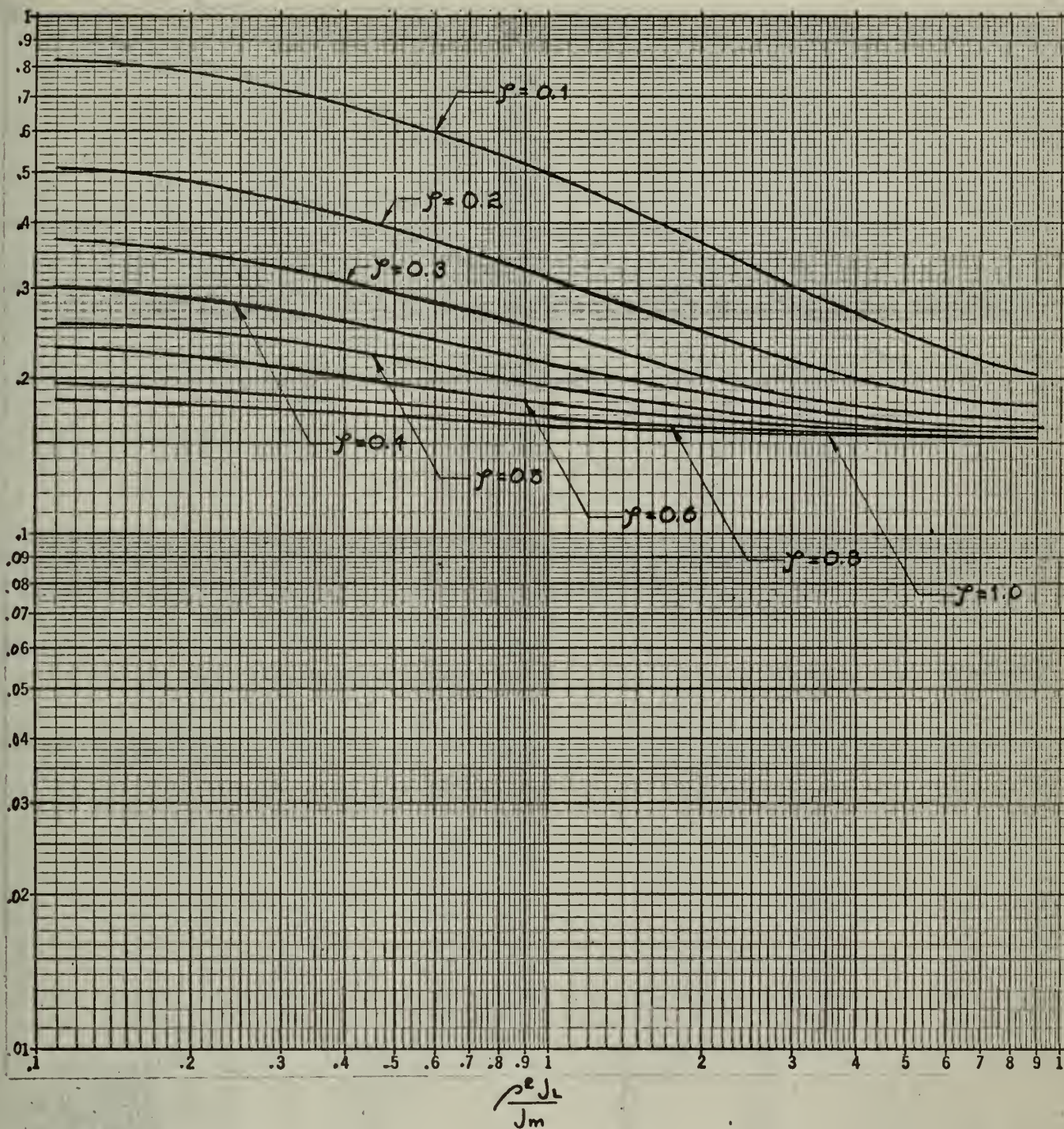


Fig. 3

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0$$

$$e = 0$$



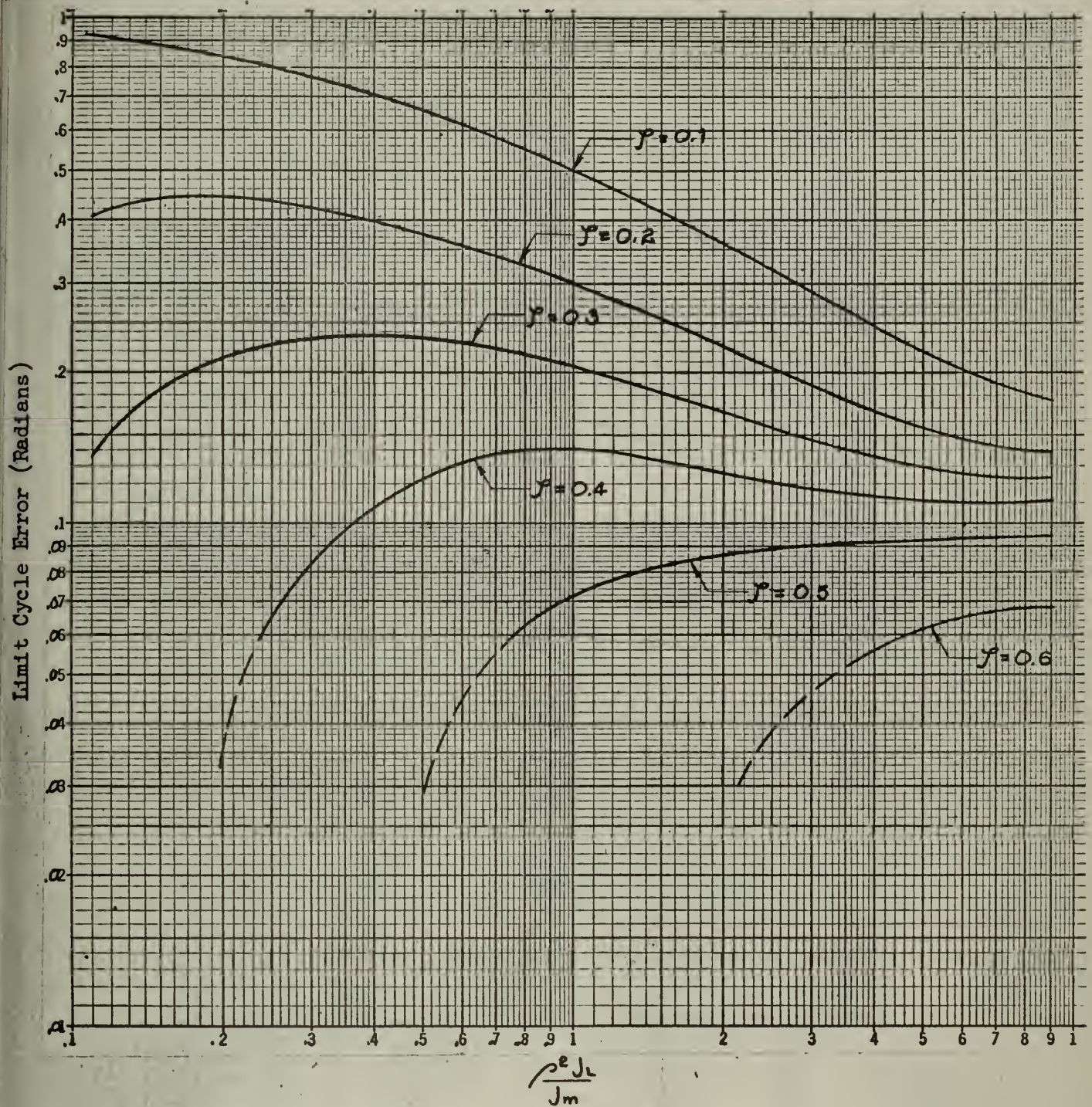


Fig. 4

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$$

$$e = 0$$

No limit cycle exists for  $\gamma \geq 0.8$



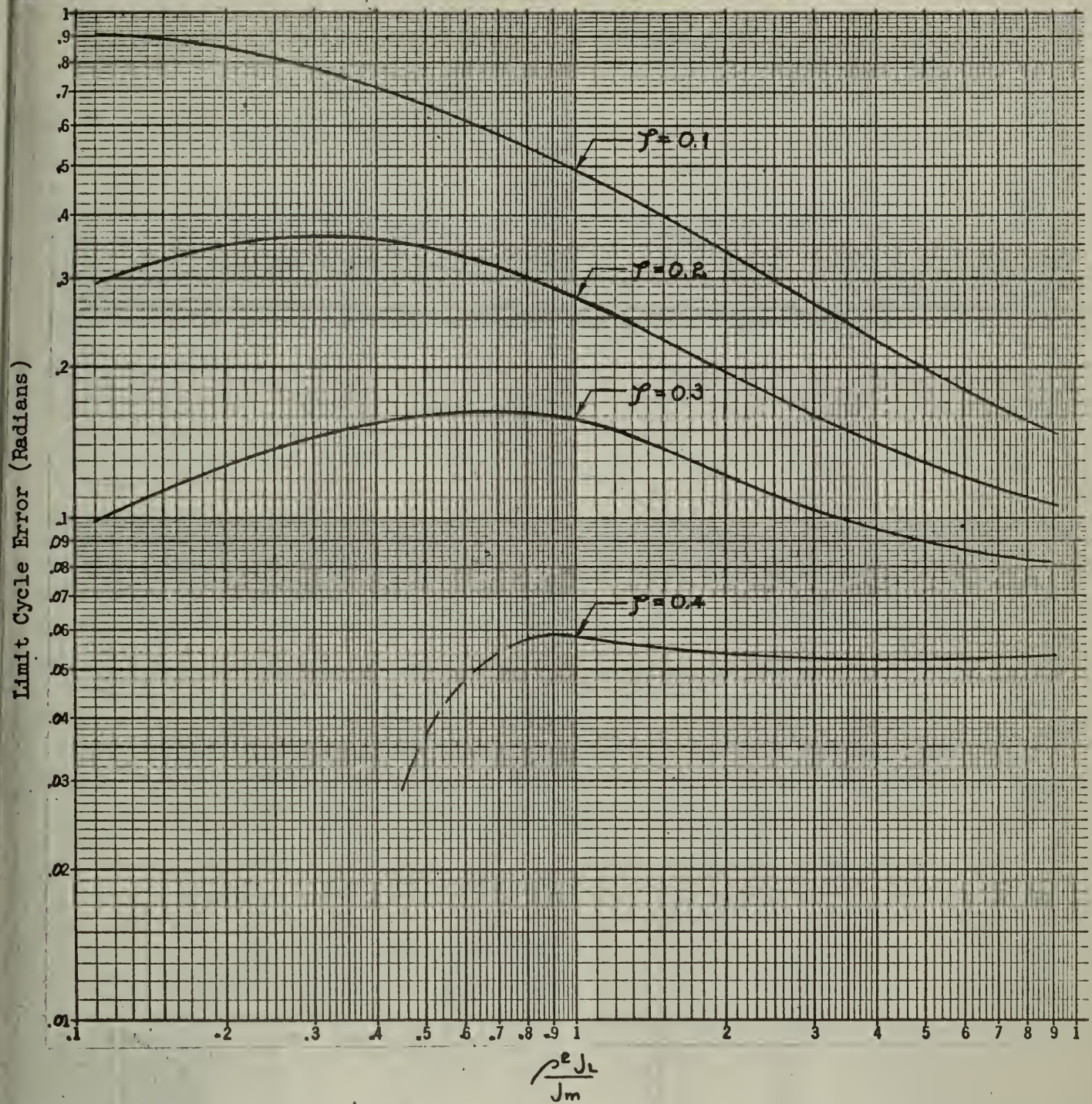


Fig. 5

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.4$$

$$e = 0$$

No limit cycle exists for  $\gamma \geq 0.5$



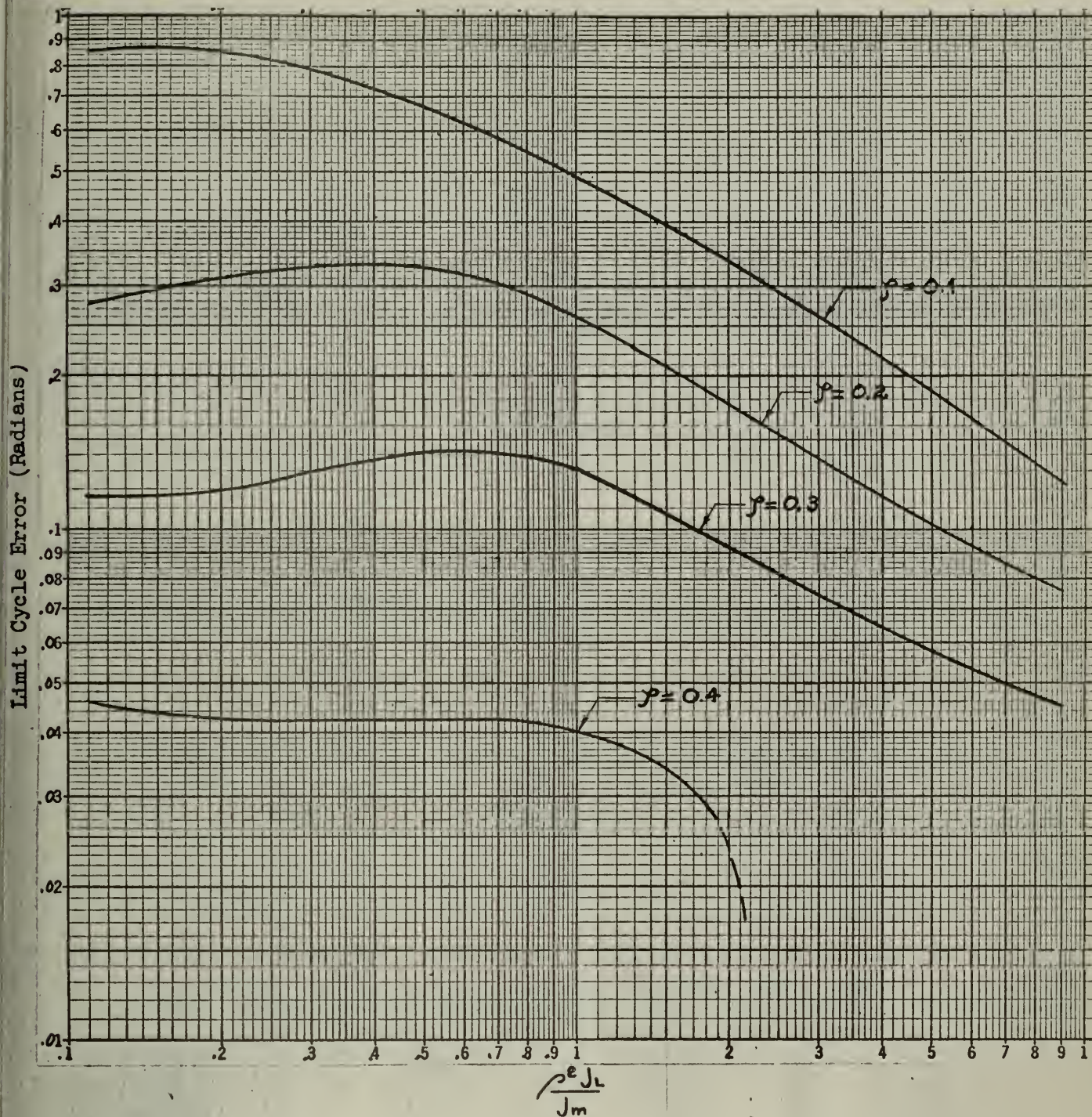


Fig. 6

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.6$$

$$e = 0$$

No limit cycle exists for  $\zeta \geq 0.5$



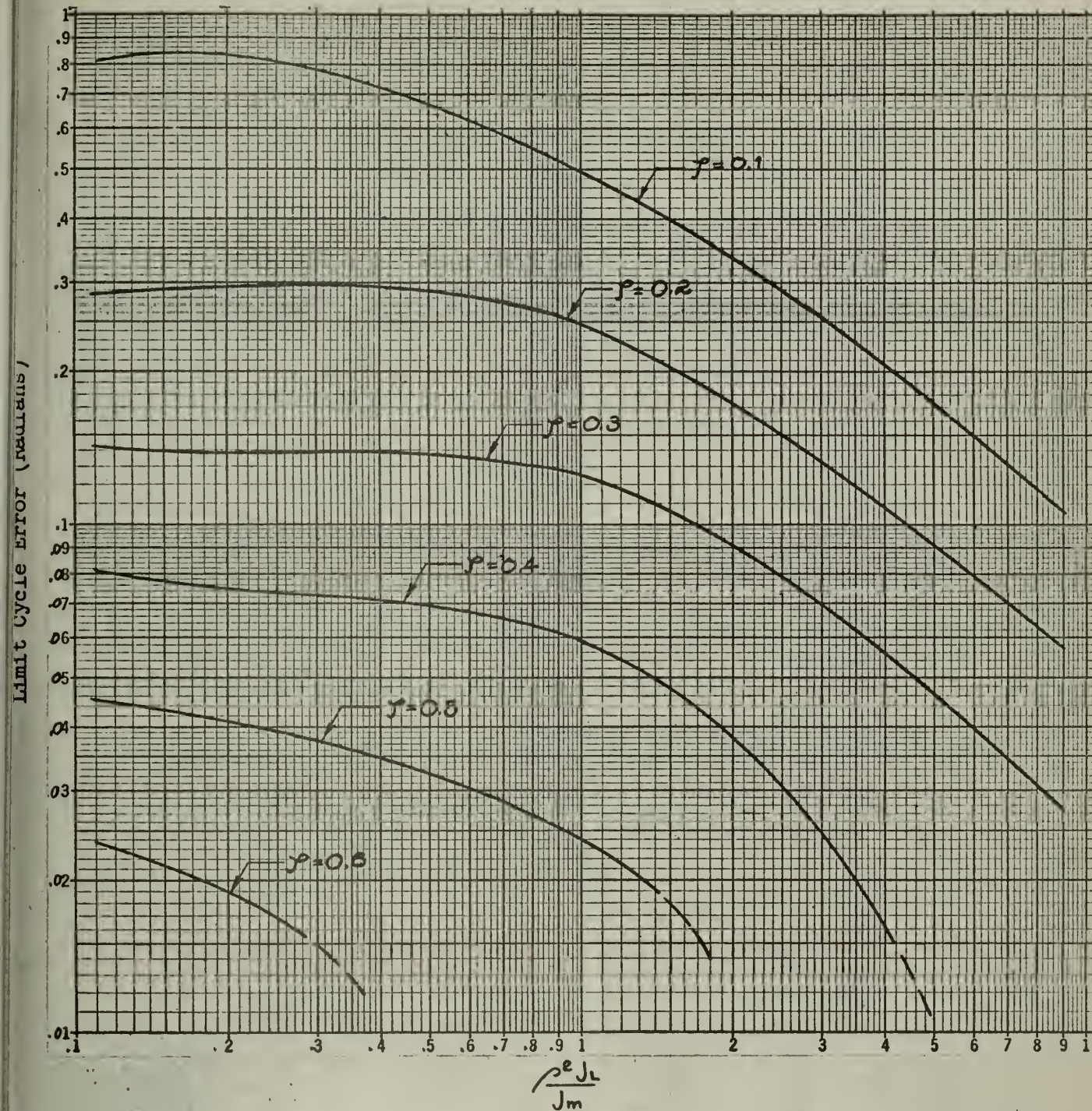


Fig. 7

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F\tau} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.8$$

$$e = 0$$

No limit cycle exists for  $\gamma \geq 0.8$



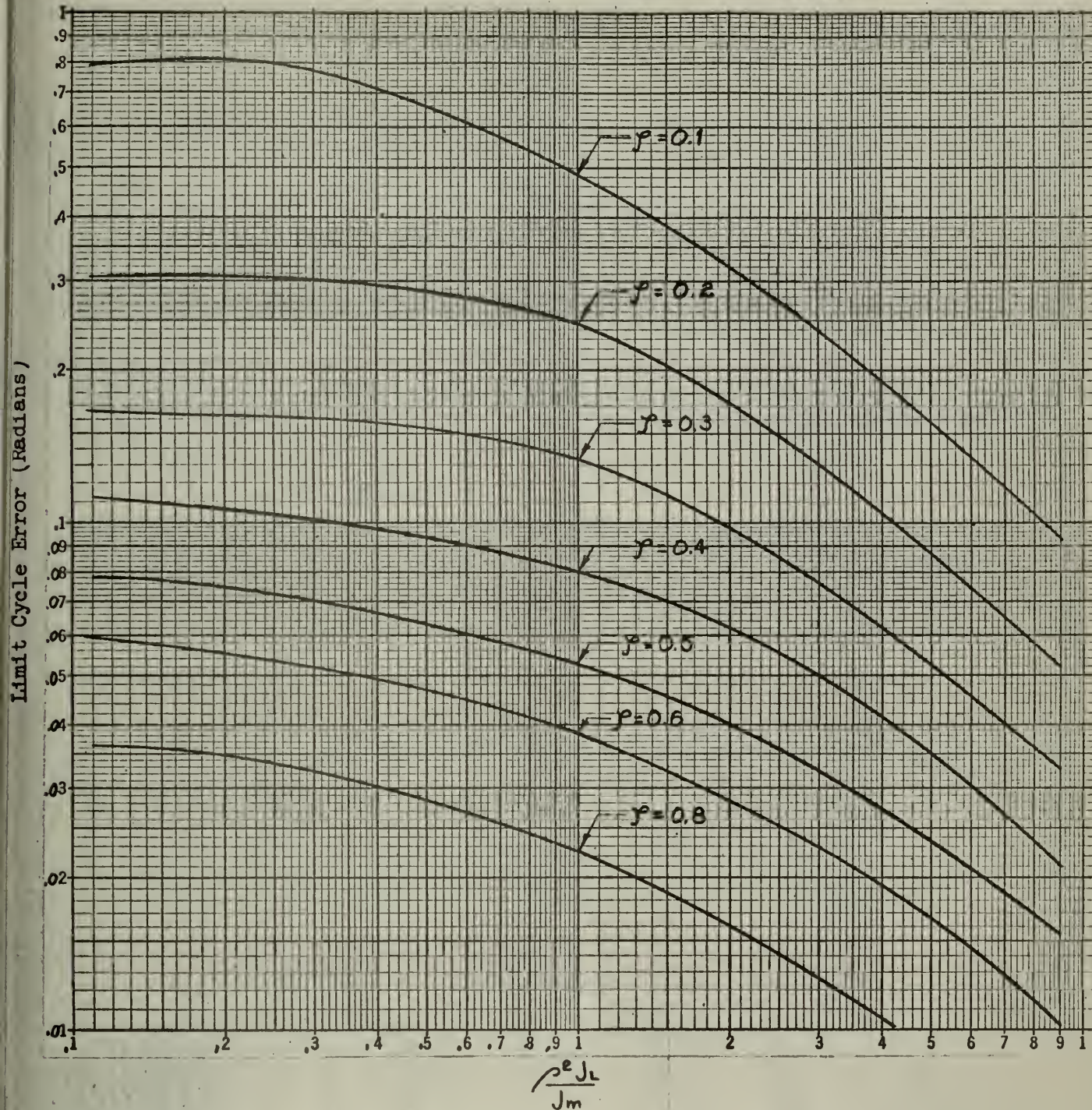


Fig. 8

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 1.0$$

$$e = 0$$

No limit cycle exists for  $\gamma \geq 1.0$



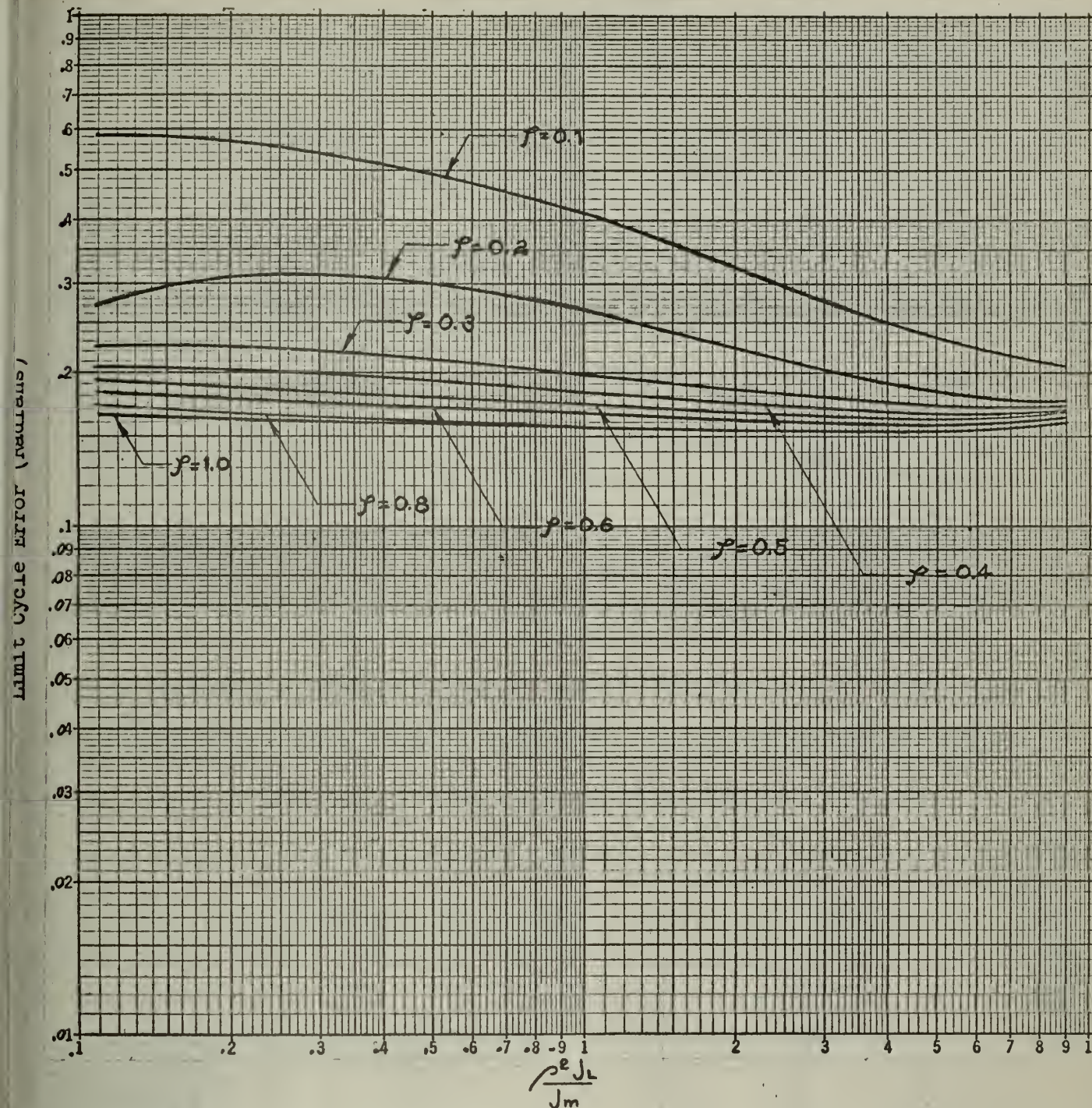


Fig. 9

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_r} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0$$

$$e = 0.6$$



Limit Cycle Error (Radians)

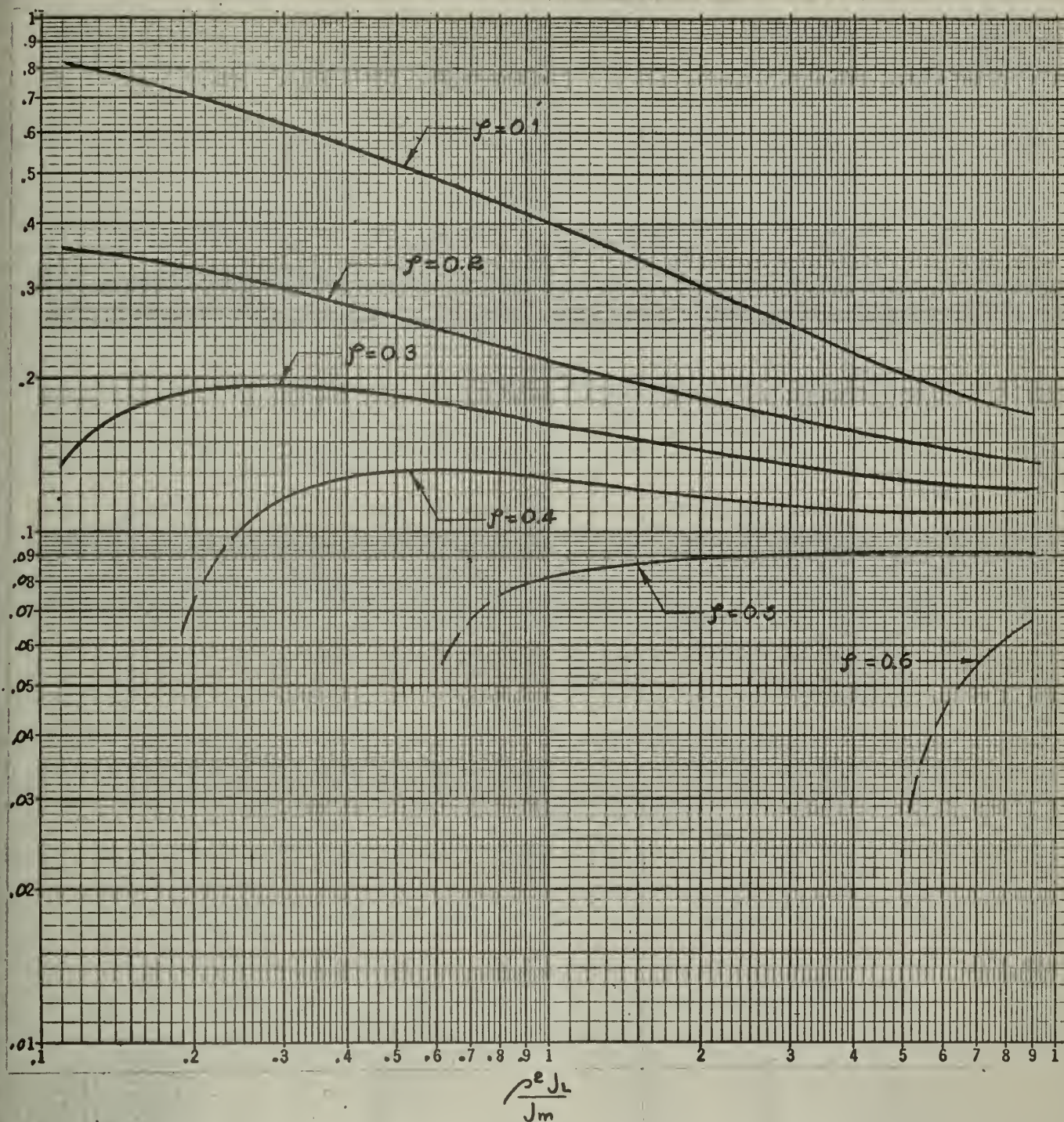


Fig. 10

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$$

$$e = 0.6$$

No limit cycle exists for  $\gamma \geq 0.8$



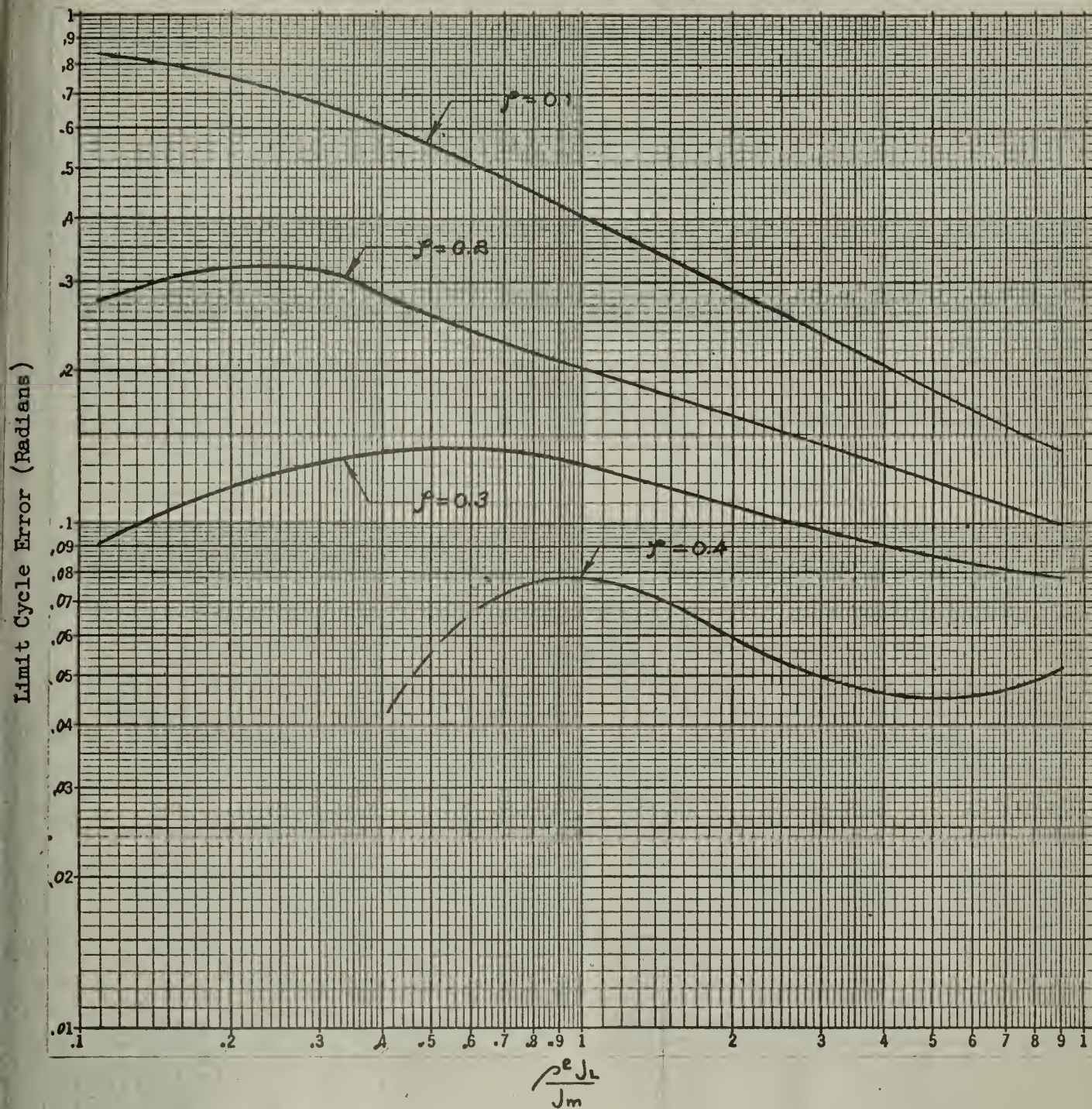


Fig. 11

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F\tau} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.4$$

$$e = 0.6$$

No limit cycle exists for  $\gamma \geq 0.5$



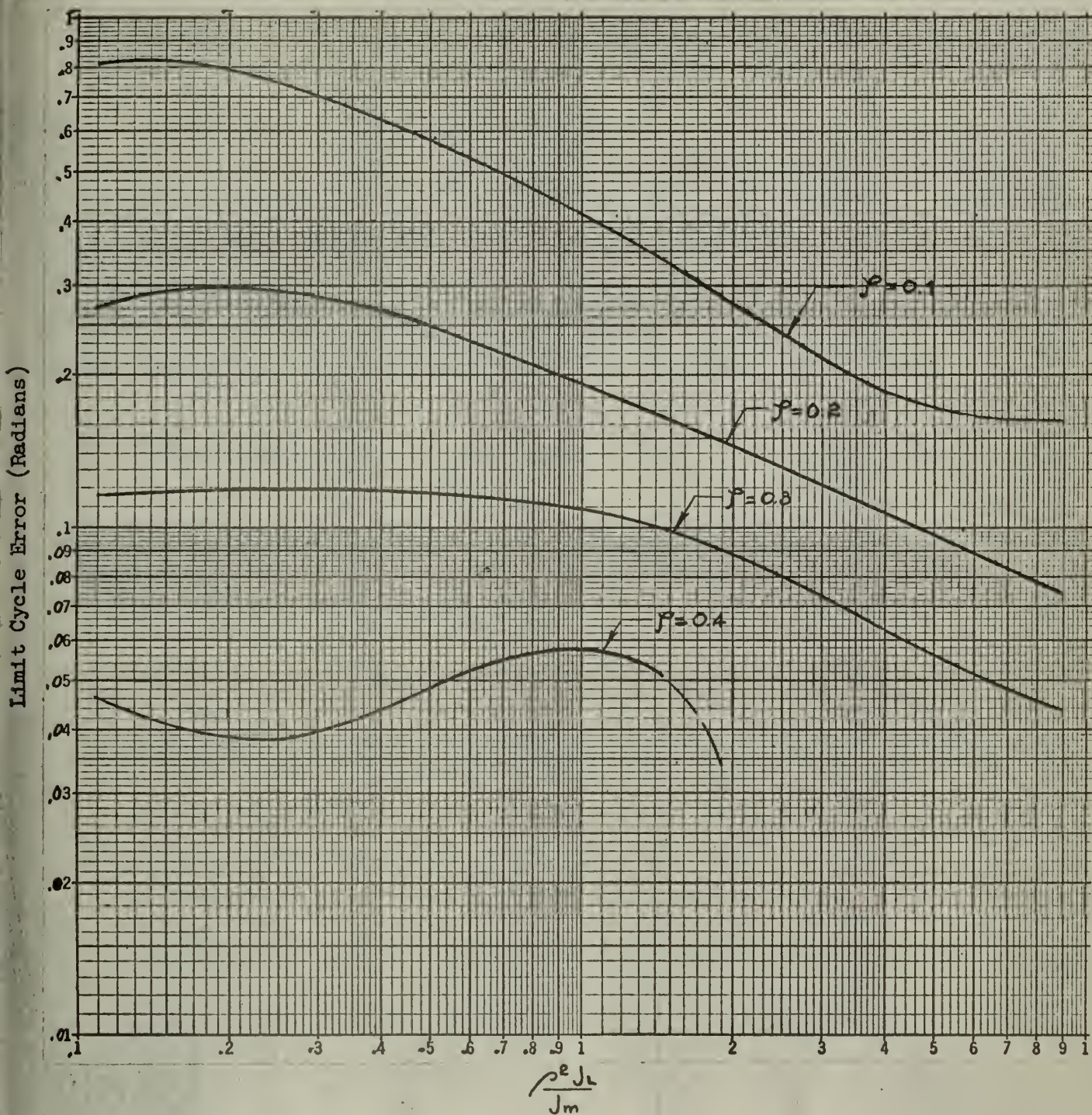


Fig. 12

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.6$$

$$e = 0.6$$

No limit cycle exists for  $\gamma \geq 0.5$



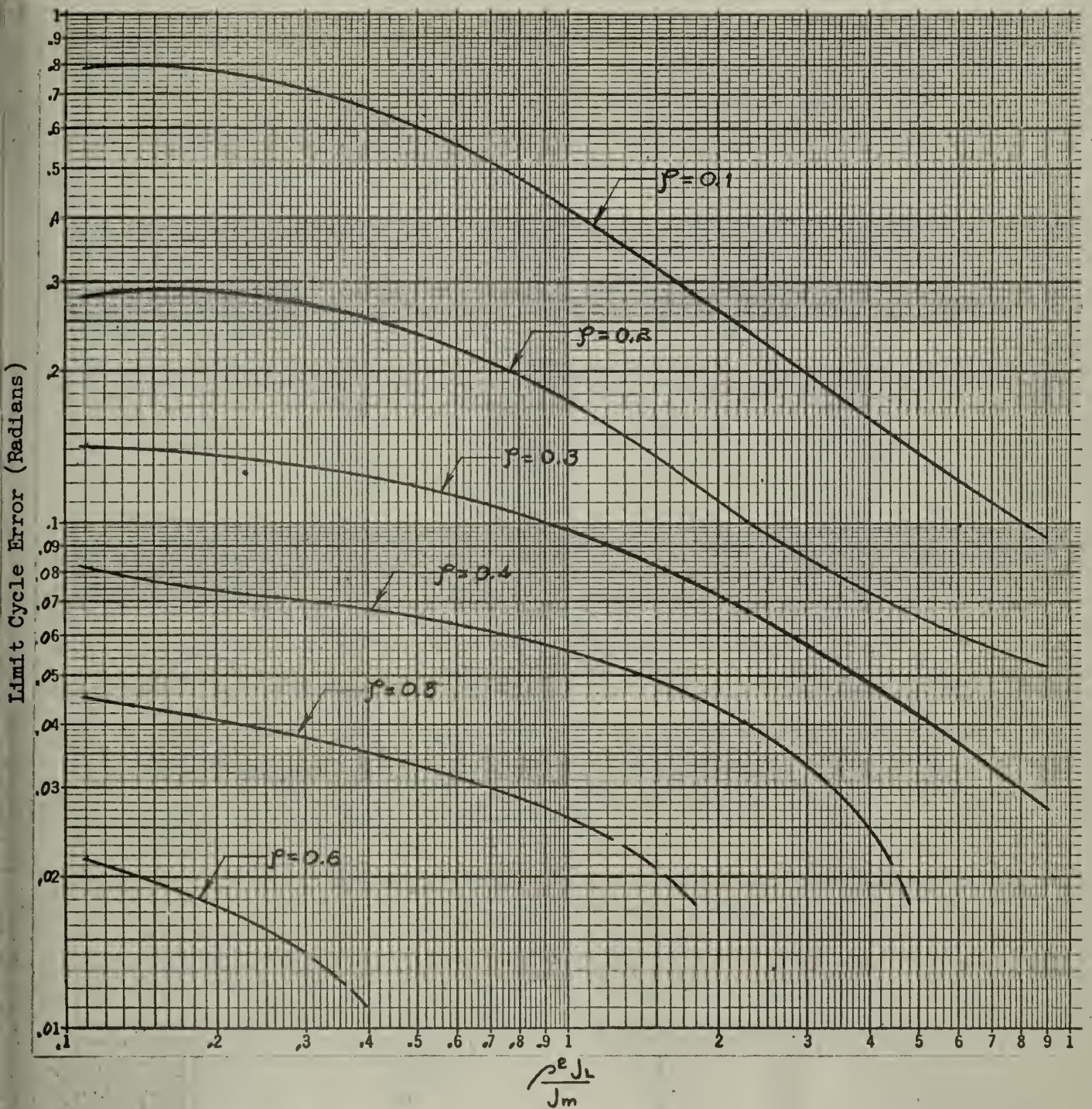


Fig. 13

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.8$$

$$e = 0.6$$

No limit cycle exists for  $\rho \geq 0.8$



Limit Cycle Error (Radians)

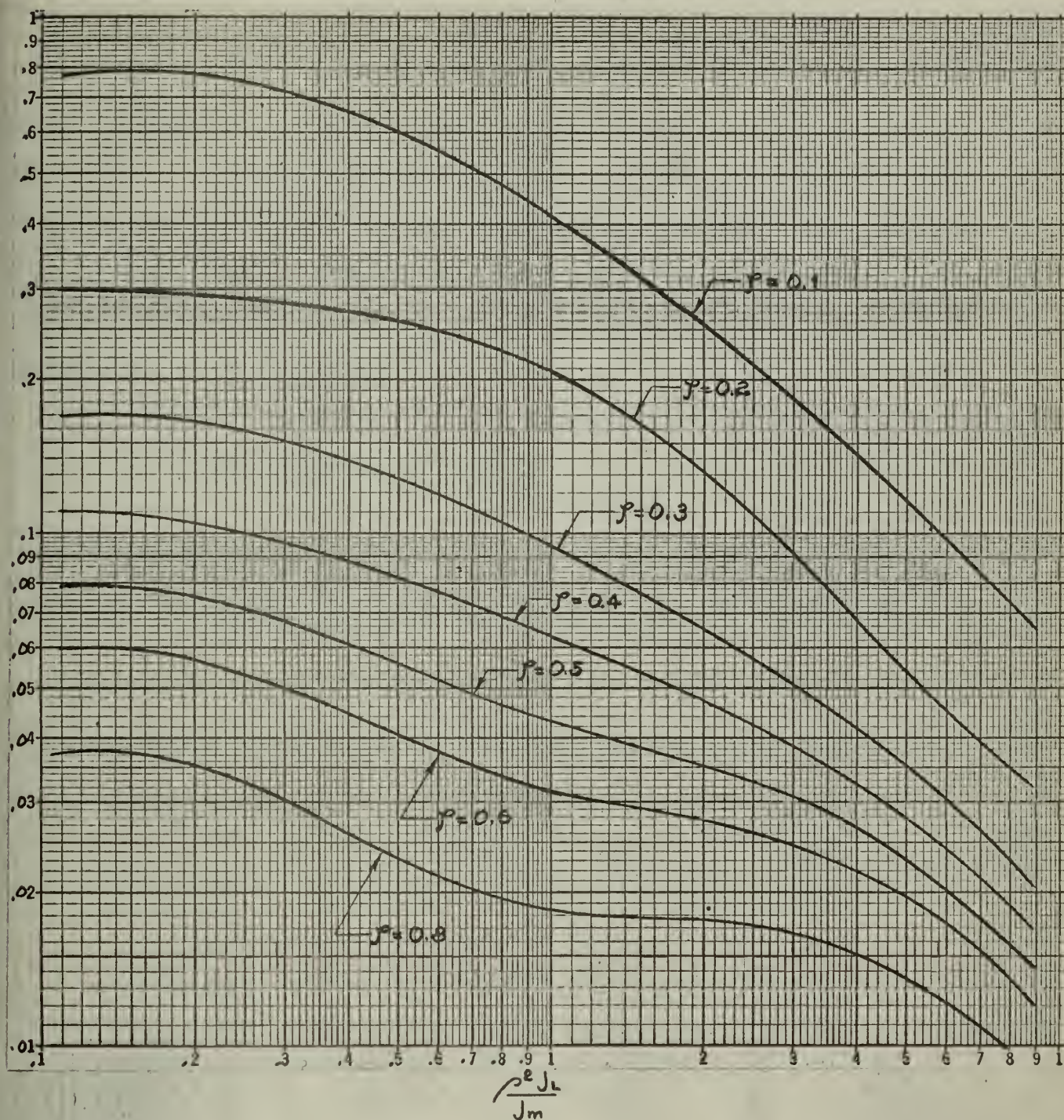


Fig. 14

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 1.0$$

$$e = 0.6$$

No limit cycle exists for  $p \geq 1.0$



Limit Cycle Error (Radians)

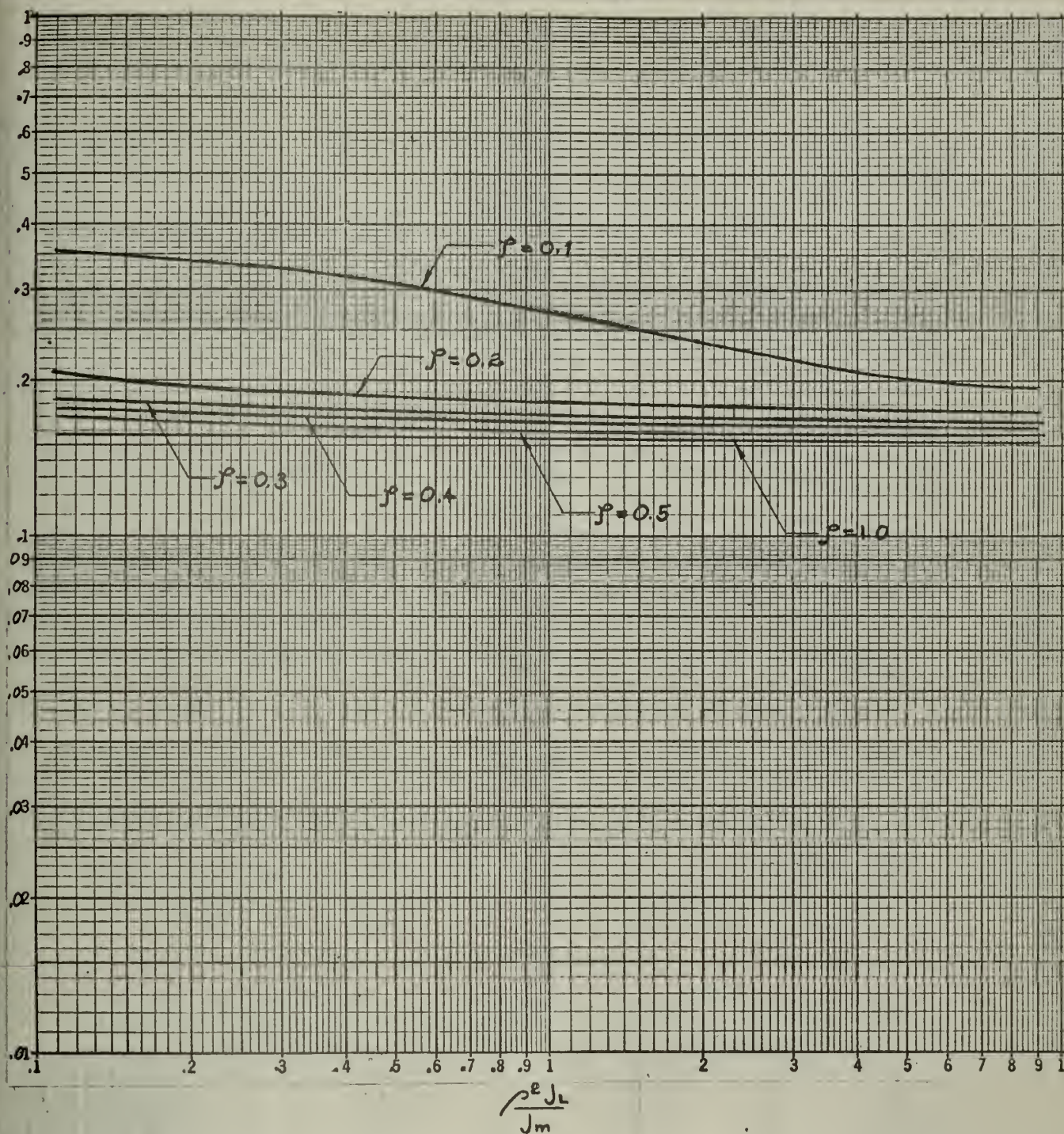


Fig. 15

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0$$

$$e = 0.8$$



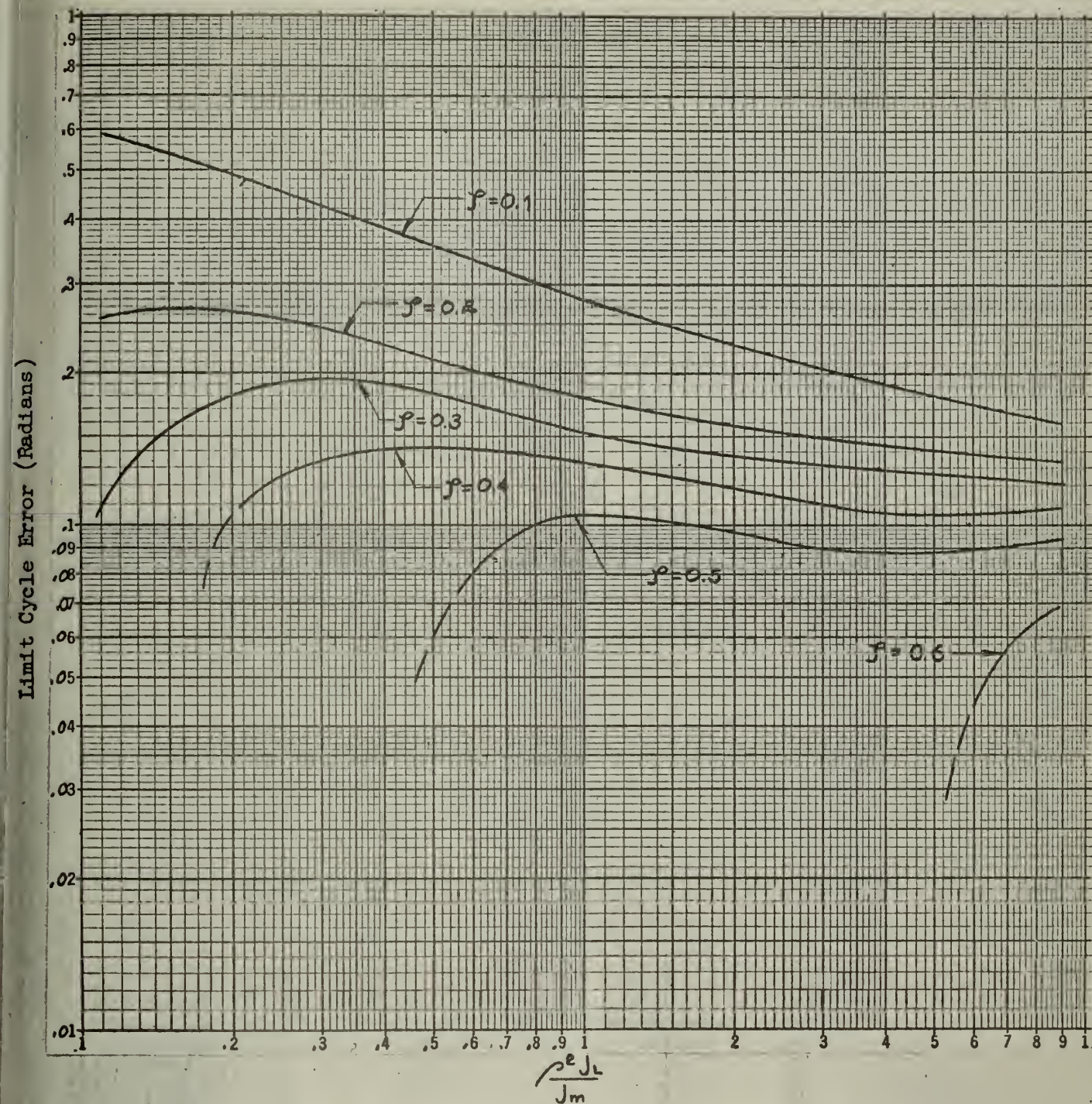


Fig. 16

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$$

$$e = 0.8$$

No limit cycle exists for  $\gamma \geq 0.8$



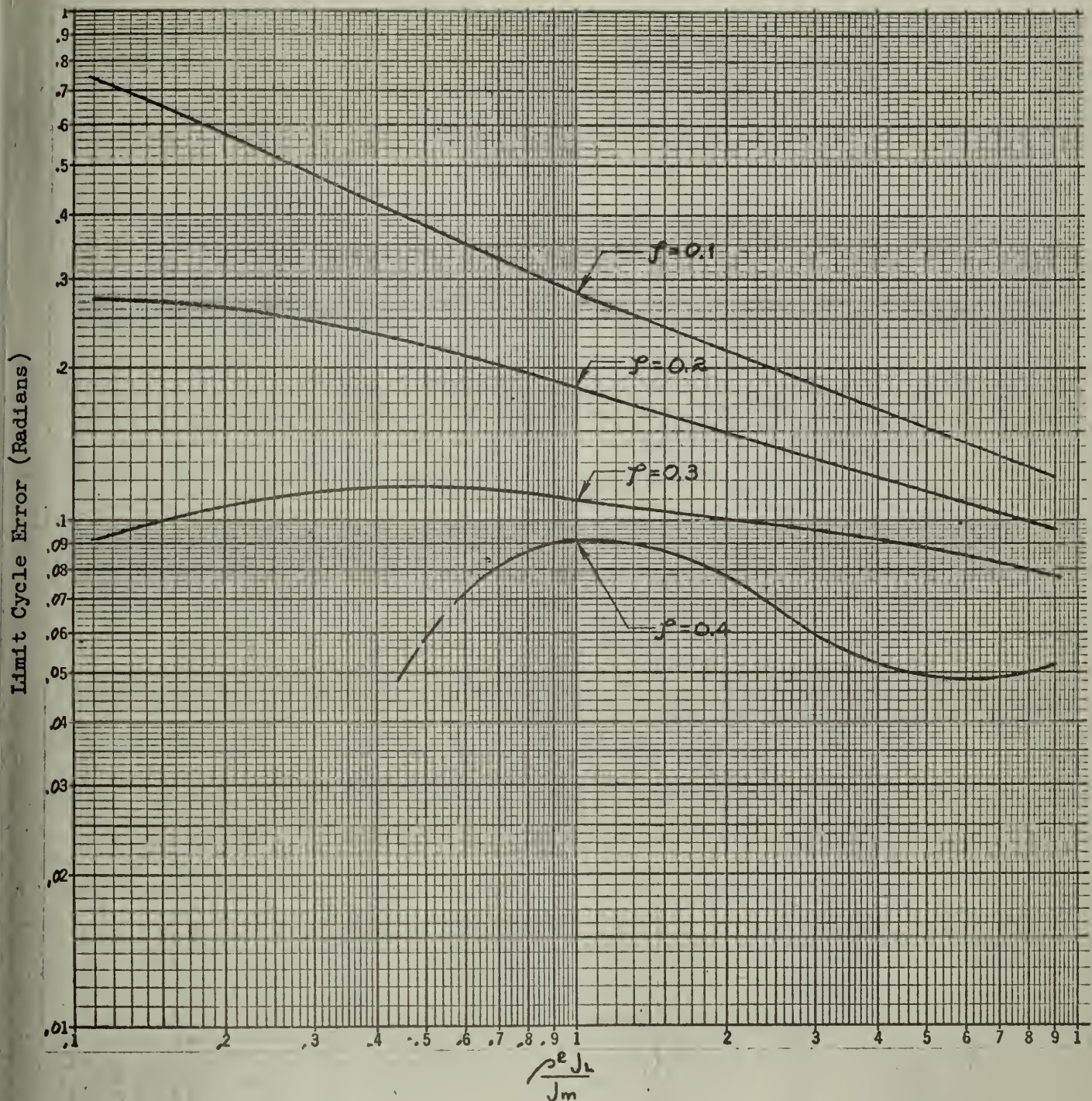


Fig. 17

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F\tau} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.4$$

$$e = 0.8$$

No limit cycle exists for  $\rho \geq 0.5$



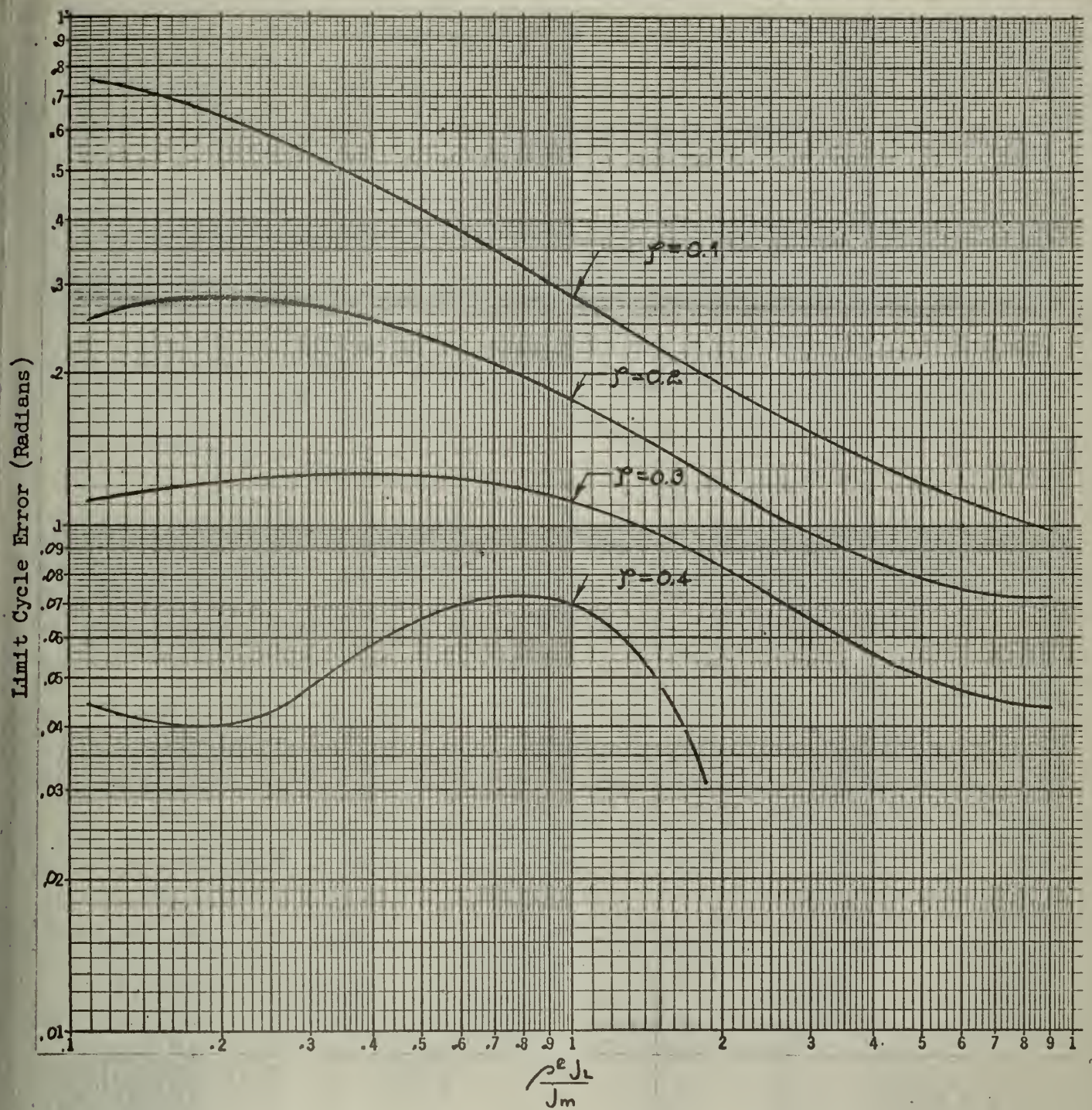


Fig. 18

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.6$$

$$e = 0.8$$

No limit cycle exists for  $\gamma \geq 0.5$



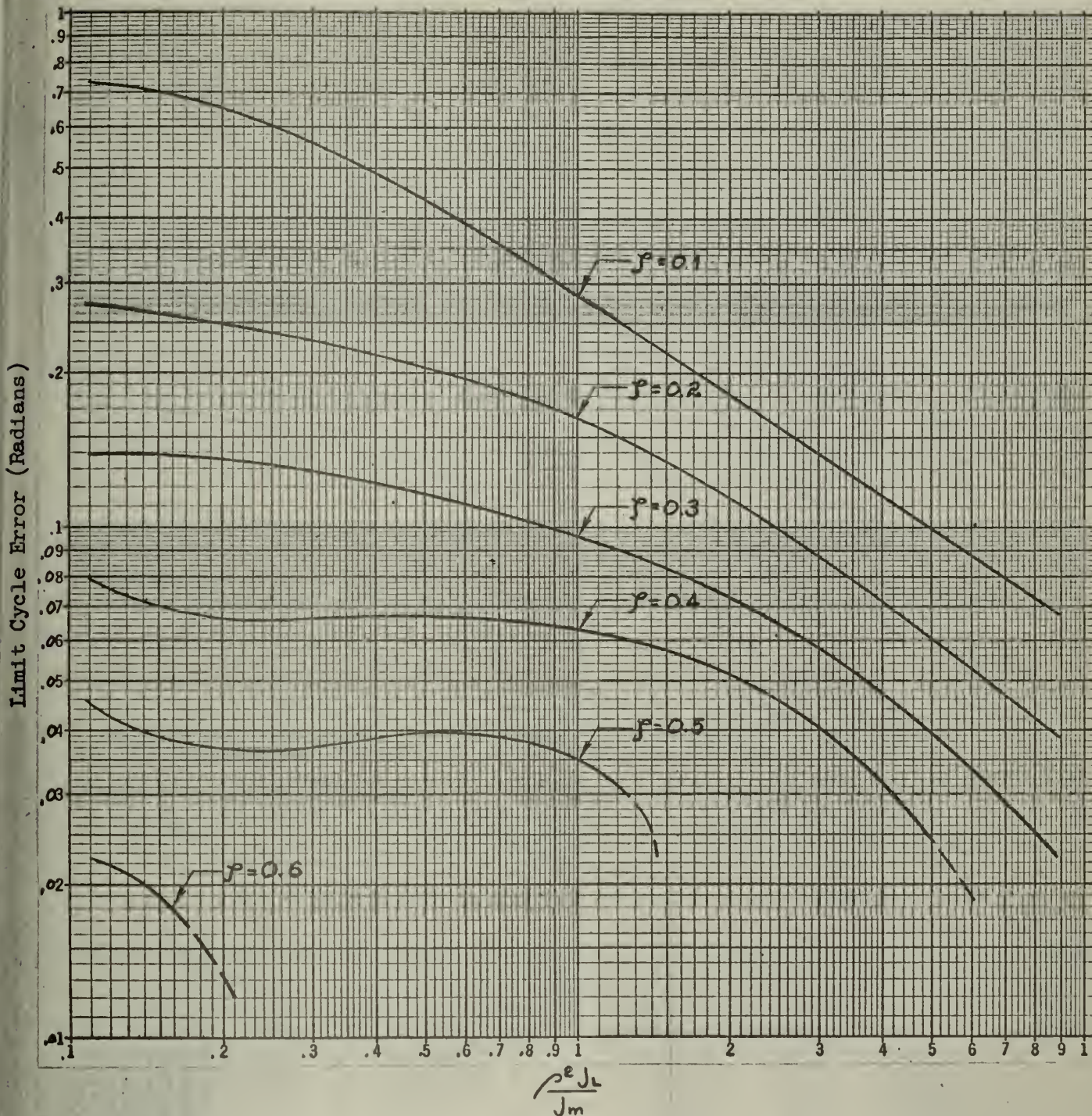


Fig. 19

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F\tau} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.8$$

$$e = 0.8$$

No limit cycle exists for  $\rho \geq 0.8$



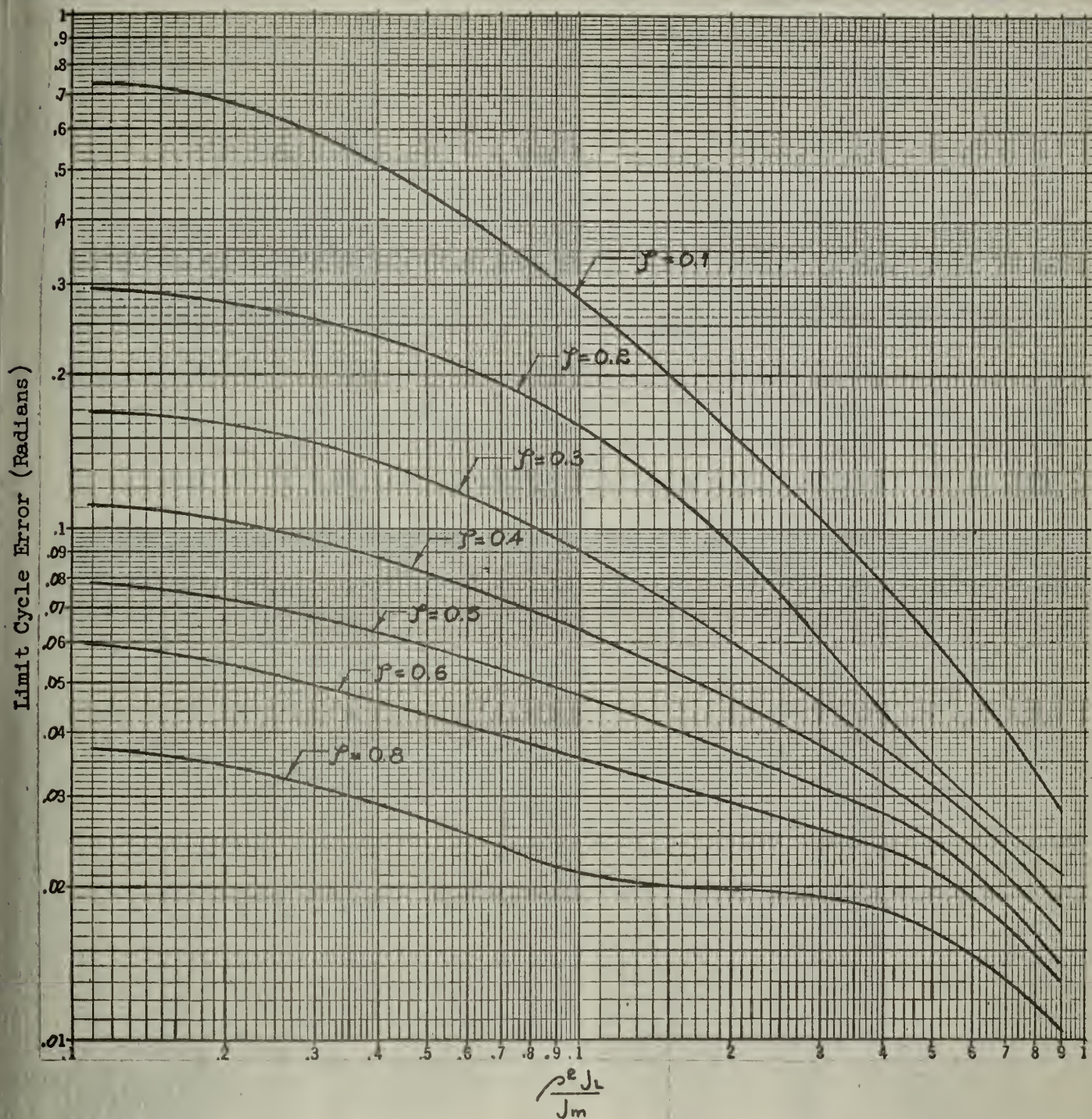


Fig. 20

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F\tau} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 1.0$$

$$e = 0.8$$

No limit cycle exists for  $\gamma \geq 1.0$



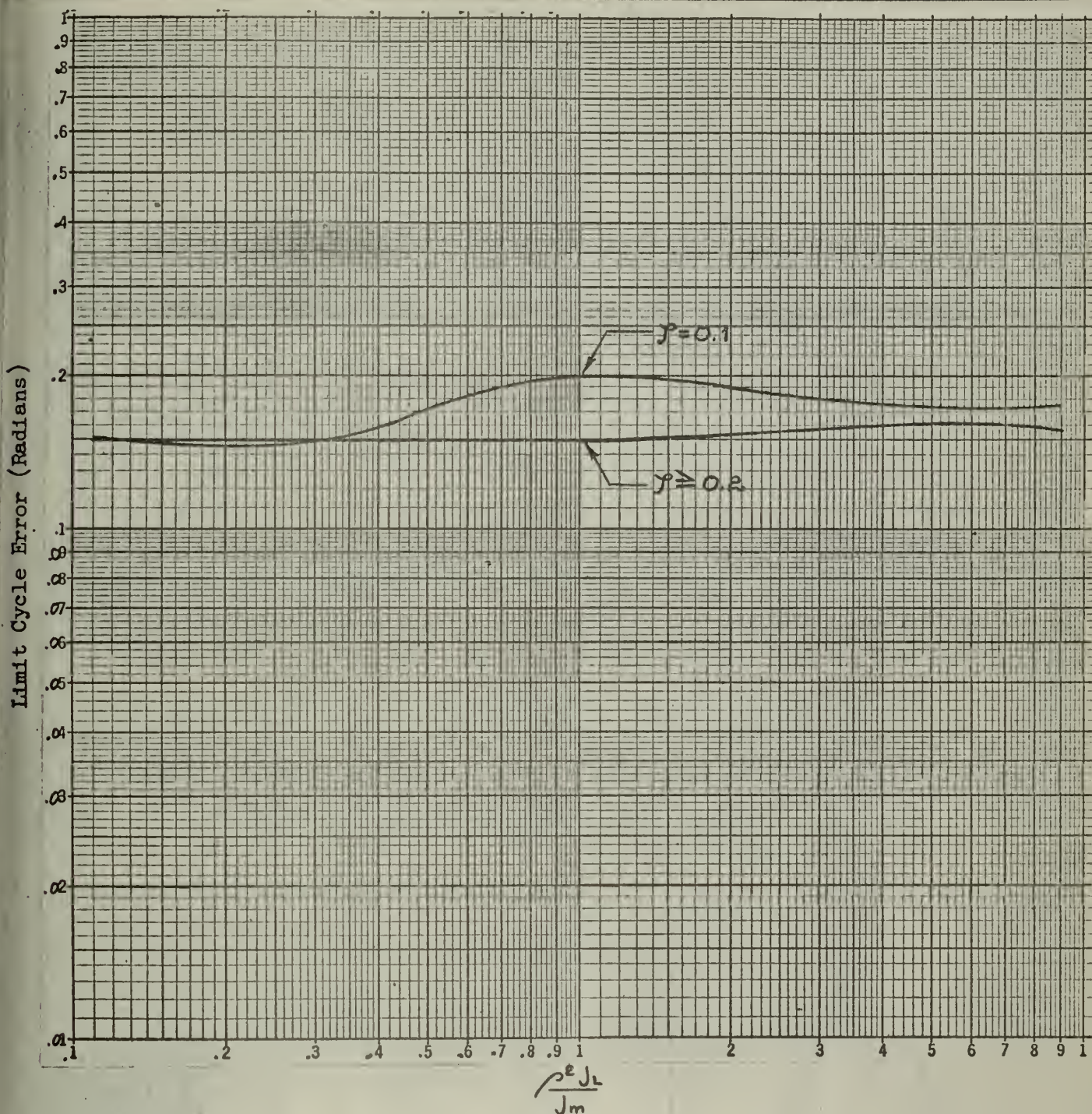


Fig. 21

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F\tau} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0$$

$$e = 1.0$$



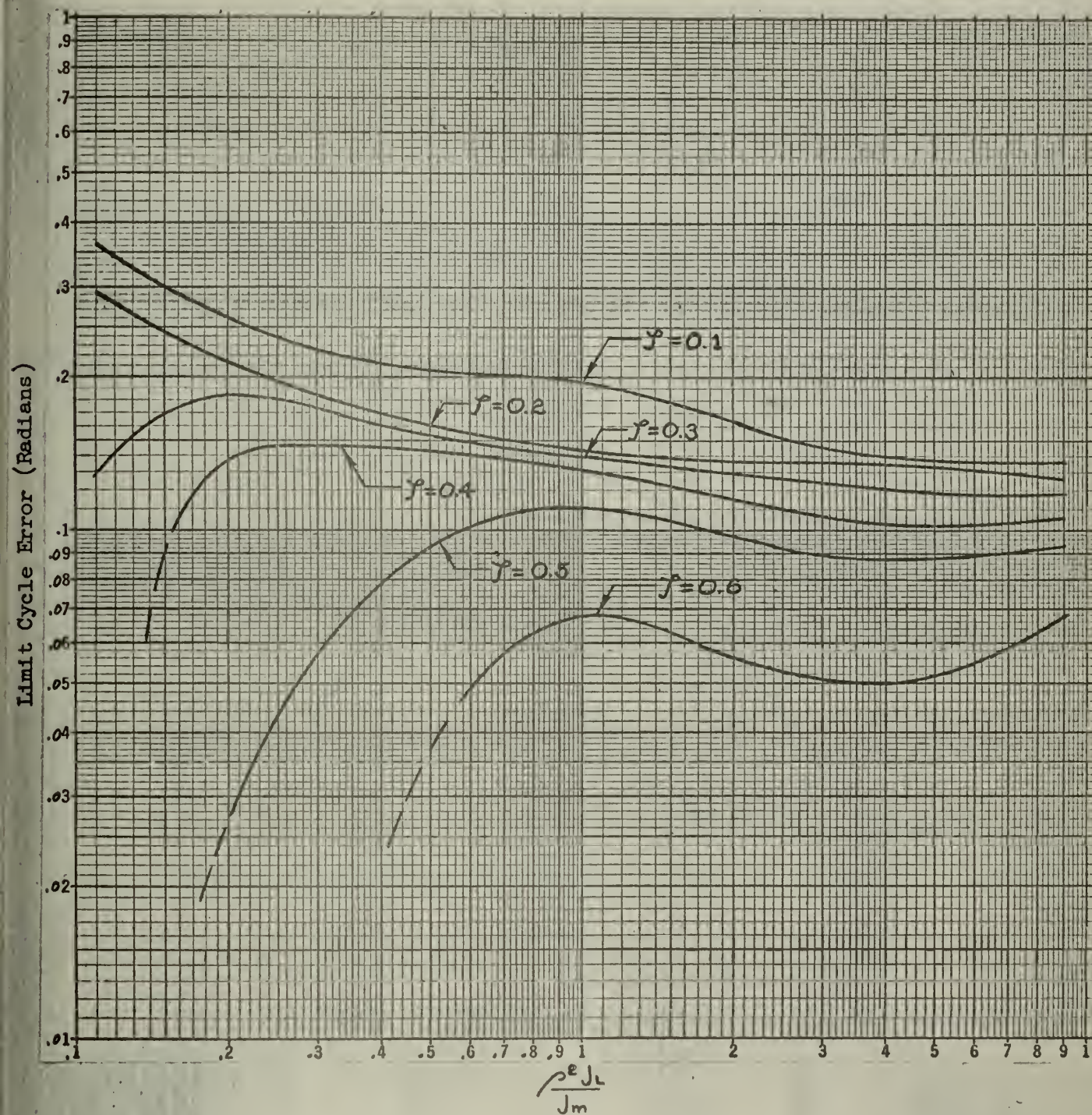


Fig. 22

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.2$$

$$e = 1.0$$

No limit cycle exists for  $\gamma \geq 0.8$



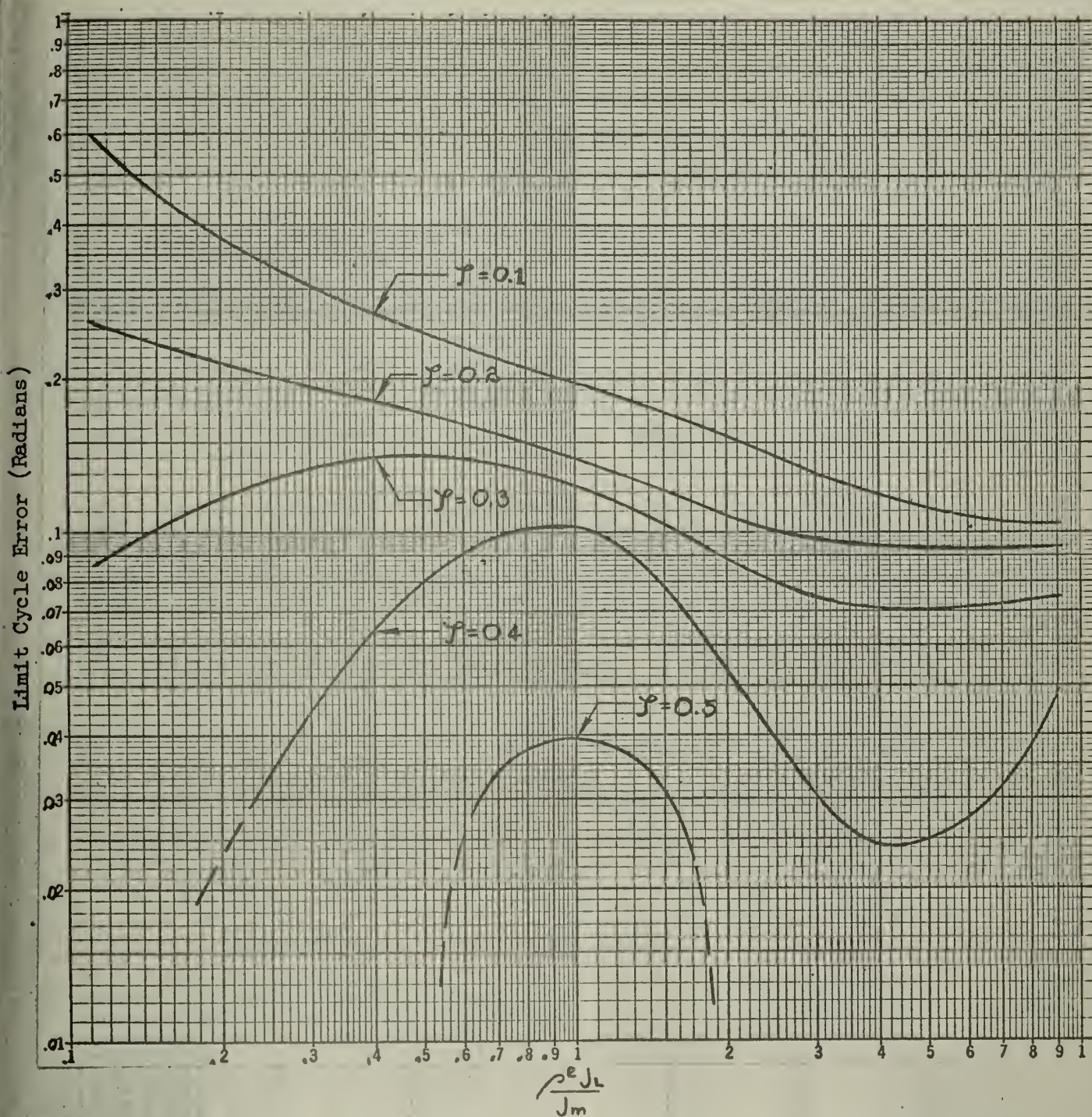


Fig. 23

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.4$$

$$e = 1.0$$

No limit cycle exists for  $\gamma \geq 0.6$



Limit Cycle Error (Radians)

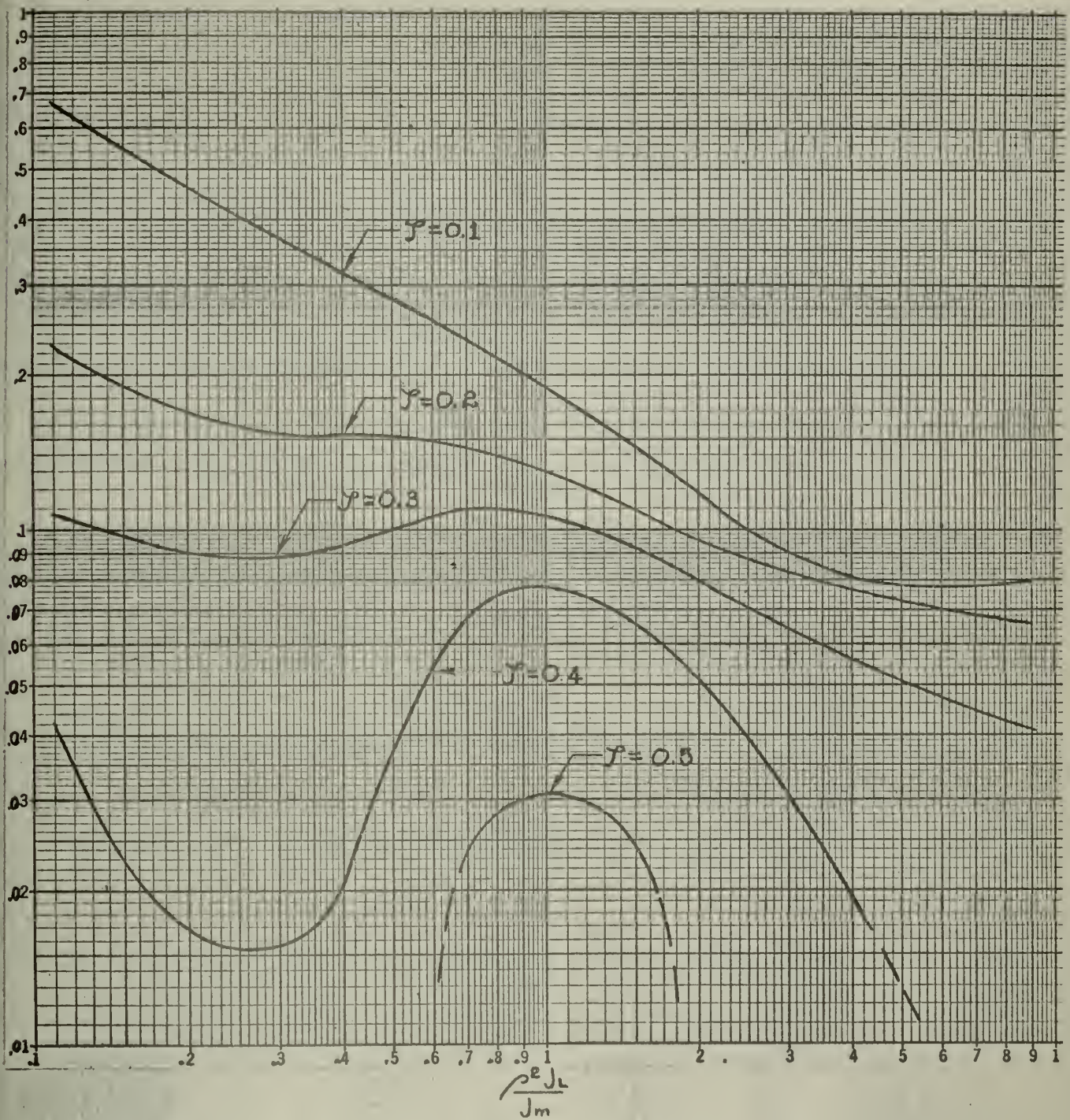


Fig. 24

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_r} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.6$$

$$e = 1.0$$

No limit cycle exists for  $\gamma \geq 0.6$



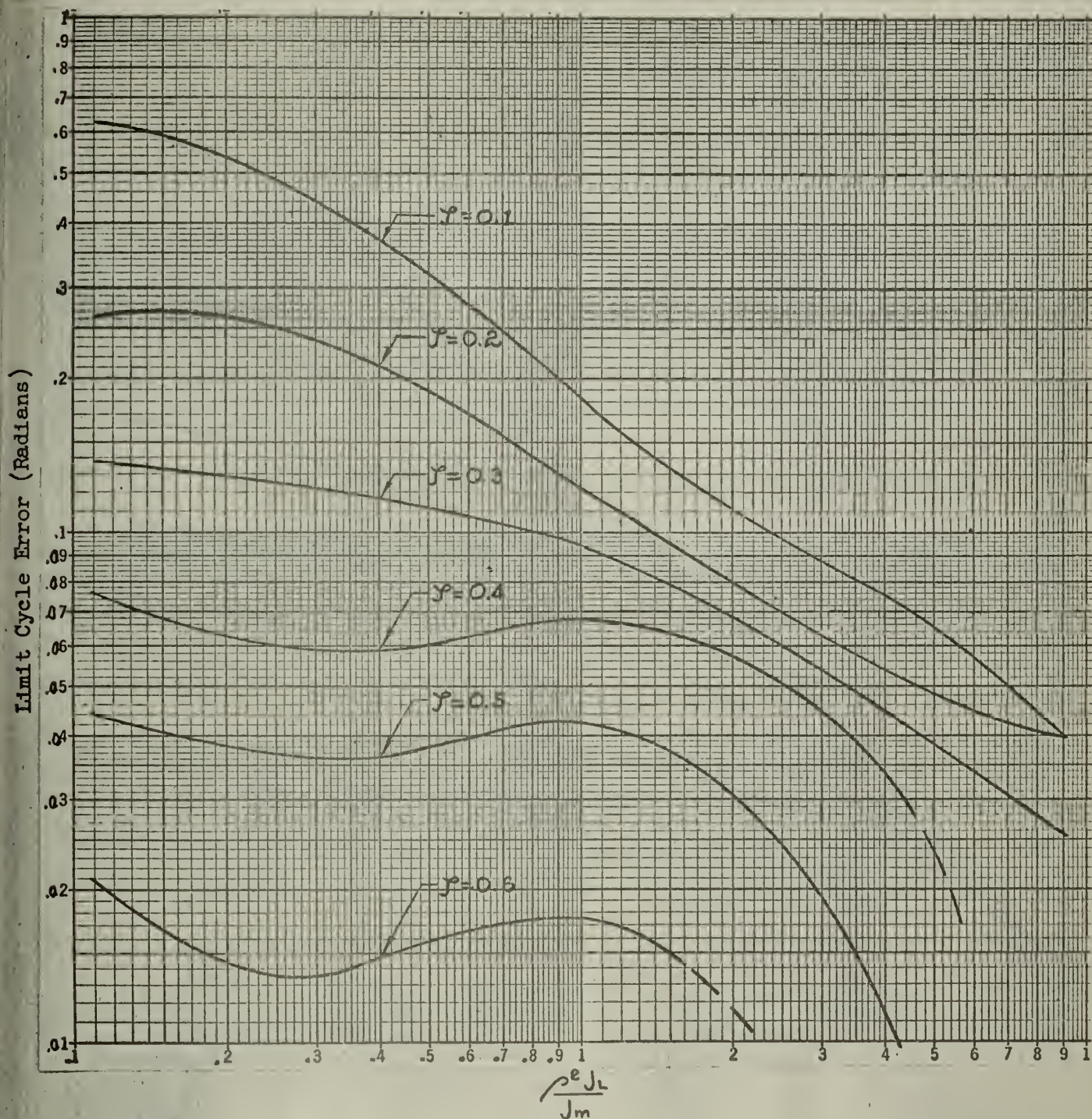


Fig. 25

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_r} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 0.8$$

$$e = 1.0$$

No limit cycle exists for  $\gamma \geq 0.8$



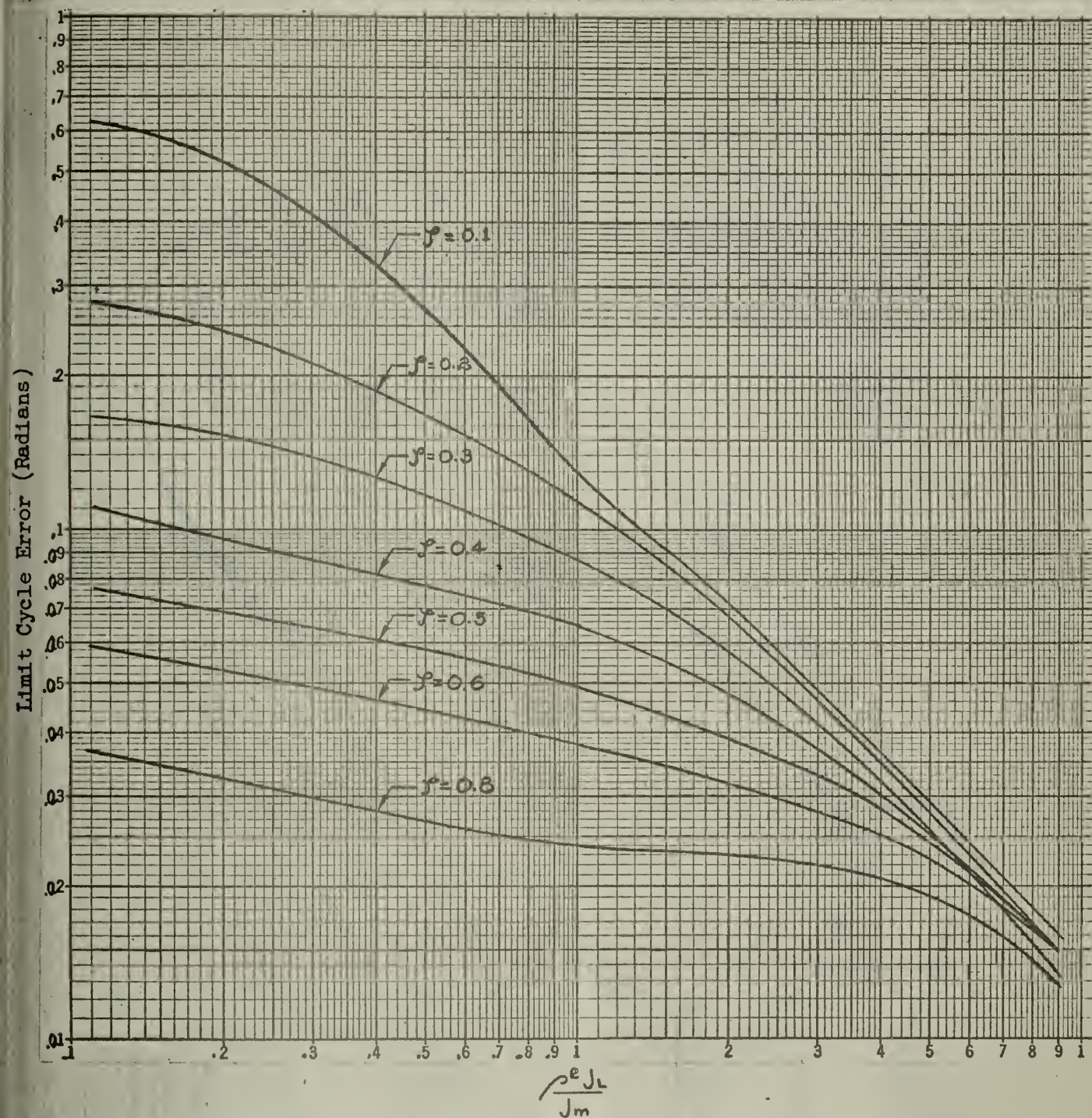


Fig. 26

Maximum Error of Limit Cycle from Unit Step Input

Limit Cycle Error (Radians)  $\left( \frac{0.3}{\Delta} \right)$  for  $\frac{\rho^2 J_L}{J_m}$  Variable

$$\frac{\rho^2 f_L}{F_T} = \frac{1}{\frac{f_m}{\rho^2 f_L} + 1} = 1.0$$

$$e = 1.0$$

No limit cycle exists for  $\gamma \geq 1.0$

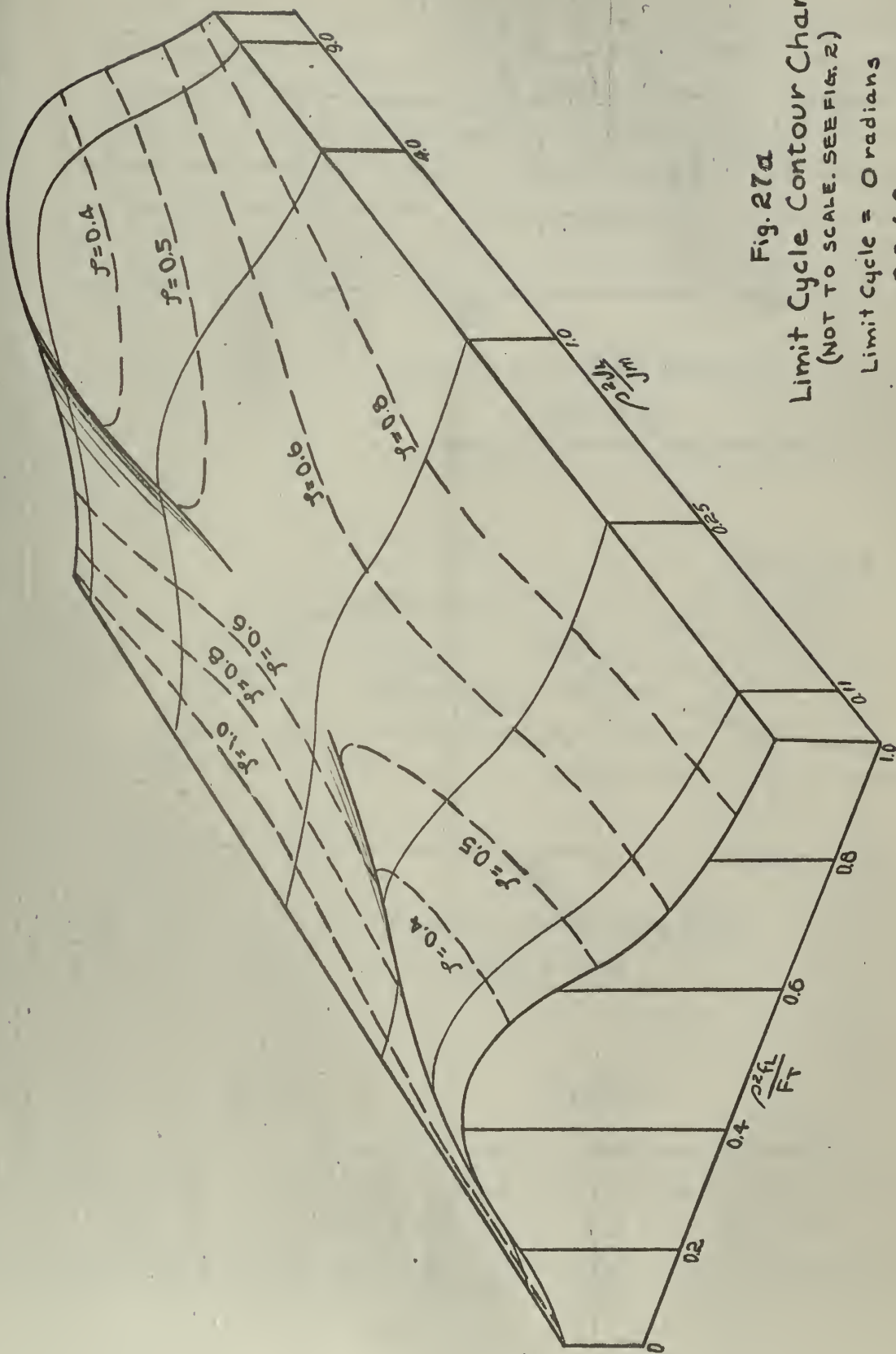
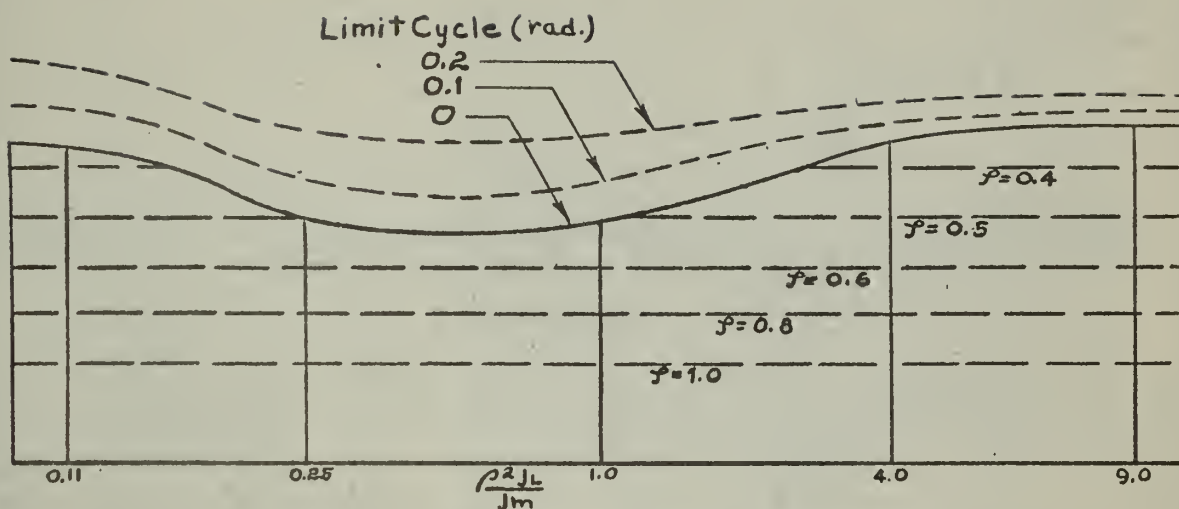


Fig. 27a  
Limit Cycle Contour Chart  
(NOT TO SCALE. SEE FIG. 2)  
Limit Cycle = 0 radians  
 $e = 1.0$

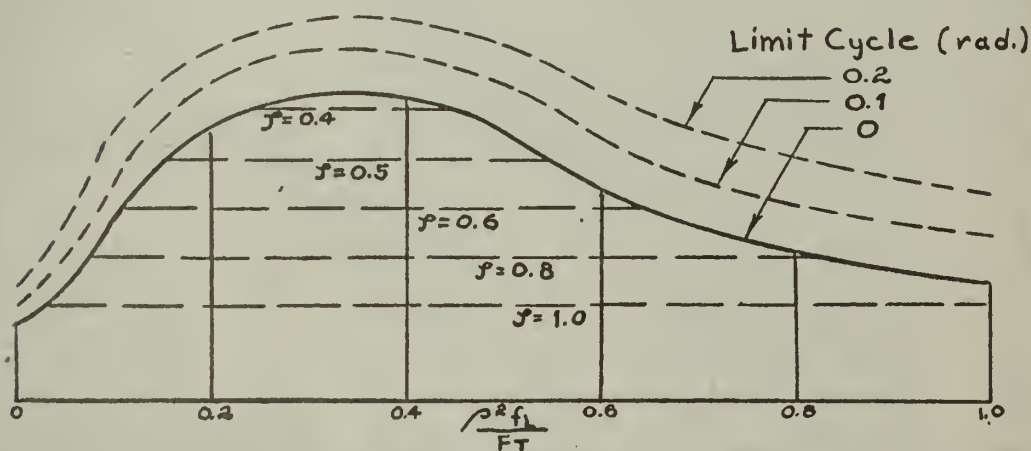




Constant  $\frac{\rho^2 f_L}{F_T}$  Profile

$$\frac{\rho^2 f_L}{F_T} = 0.6$$

(NOT TO SCALE. SEE FIG. 24)



Constant  $\frac{\rho^2 J_L}{J_m}$  Profile

$$\frac{\rho^2 J_L}{J_m} = 0.11$$

Fig. 27b

Limit Cycle Contour Chart  
Profile Views

$$c = 1.0$$

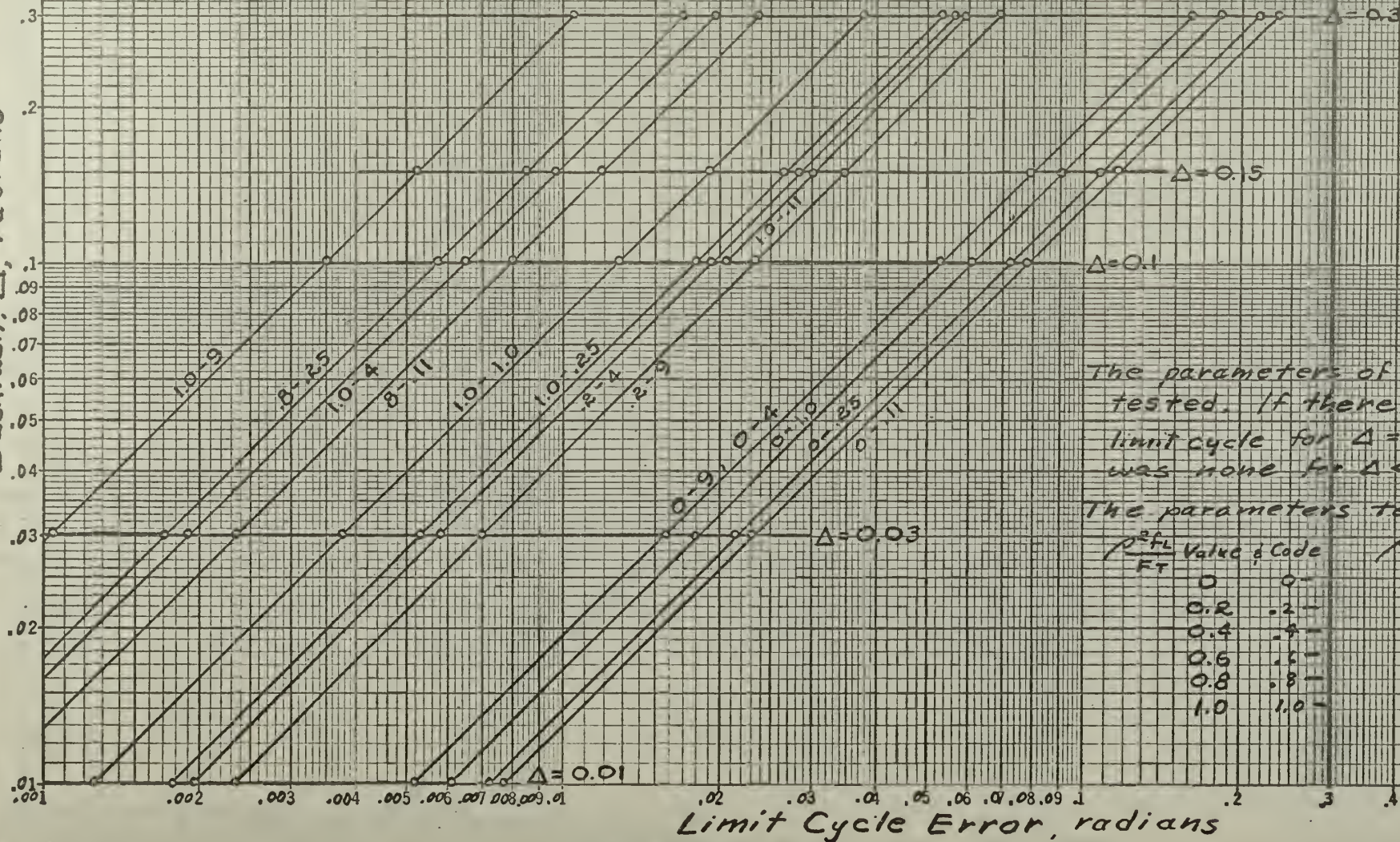
$$\Delta = 0.3$$



Backlash,  $\Delta$ , radians

Fig. 2B  
Influence of  $\Delta$  Upon  
Magnitude of Limit Cycle Error

$$p = 0.6, e = 0$$



The parameters of  $p = 0.6$  were tested. If there was no limit cycle for  $\Delta = 0.3$ , there was none for  $\Delta < 0.3$ .

The parameters tested:

$p^2 \frac{F_L}{F_T}$	Value & Code	$p^2 \frac{F_L}{F_T}$	Value & Code
0	0-	1/9	-11
0.2	.2-	2/8	-25
0.4	.4-	5/5	-10
0.6	.6-	8/2	-4
0.8	.8-	9/1	-9
1.0	1.0-		



Backlash,  $\Delta$ , radians

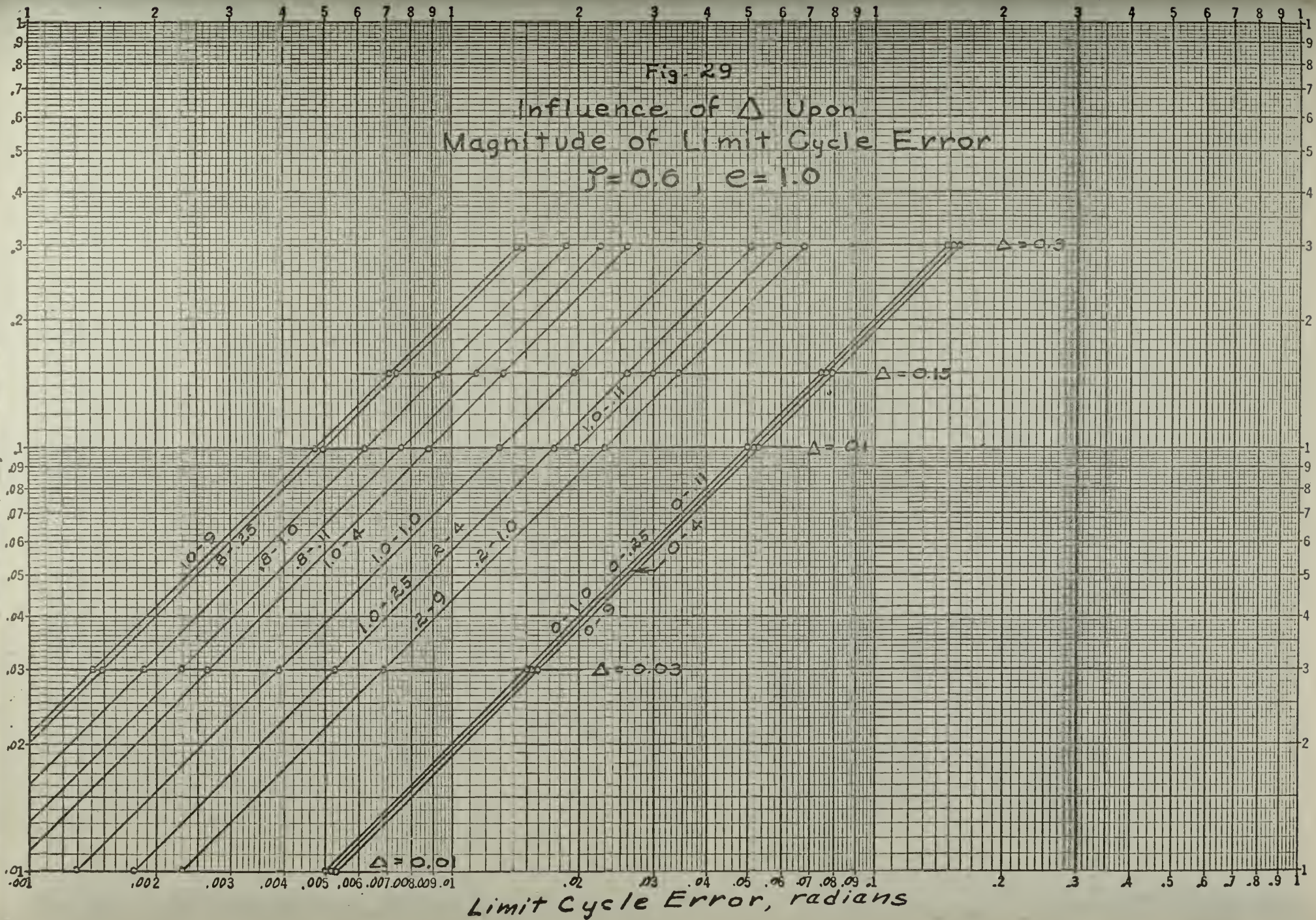




Fig. 30

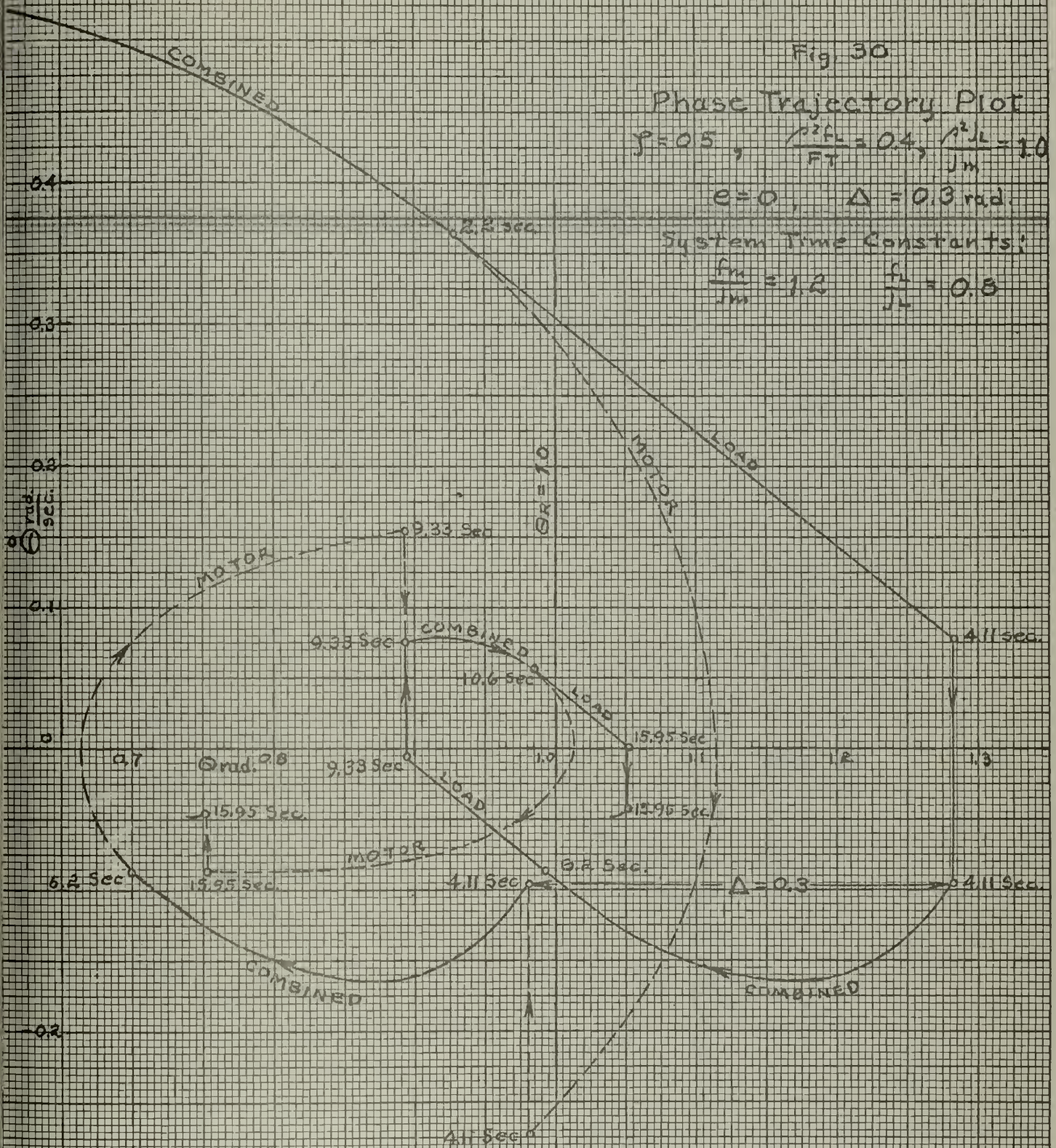
# Phase Trajectory Plot

$$\gamma = 0.5, \quad \frac{J^2 F_c}{FT} = 0.4, \quad \frac{J^2 L}{J_m} = 1.0$$

$$e = 0, \quad \Delta = 0.3 \text{ rad.}$$

System Time Constants:

$$\frac{f_m}{f_n} = 1.2, \quad \frac{f_c}{f_n} = 0.8$$



Initial Conditions:

$$\theta_R = 1.0$$

$$\theta_c = \theta_m = 0$$

$$\dot{\theta}_c = \dot{\theta}_m = 0$$

Steady State Error = 0  
(No limit cycle resulted)

See Figs. 1 and 5



Fig 31

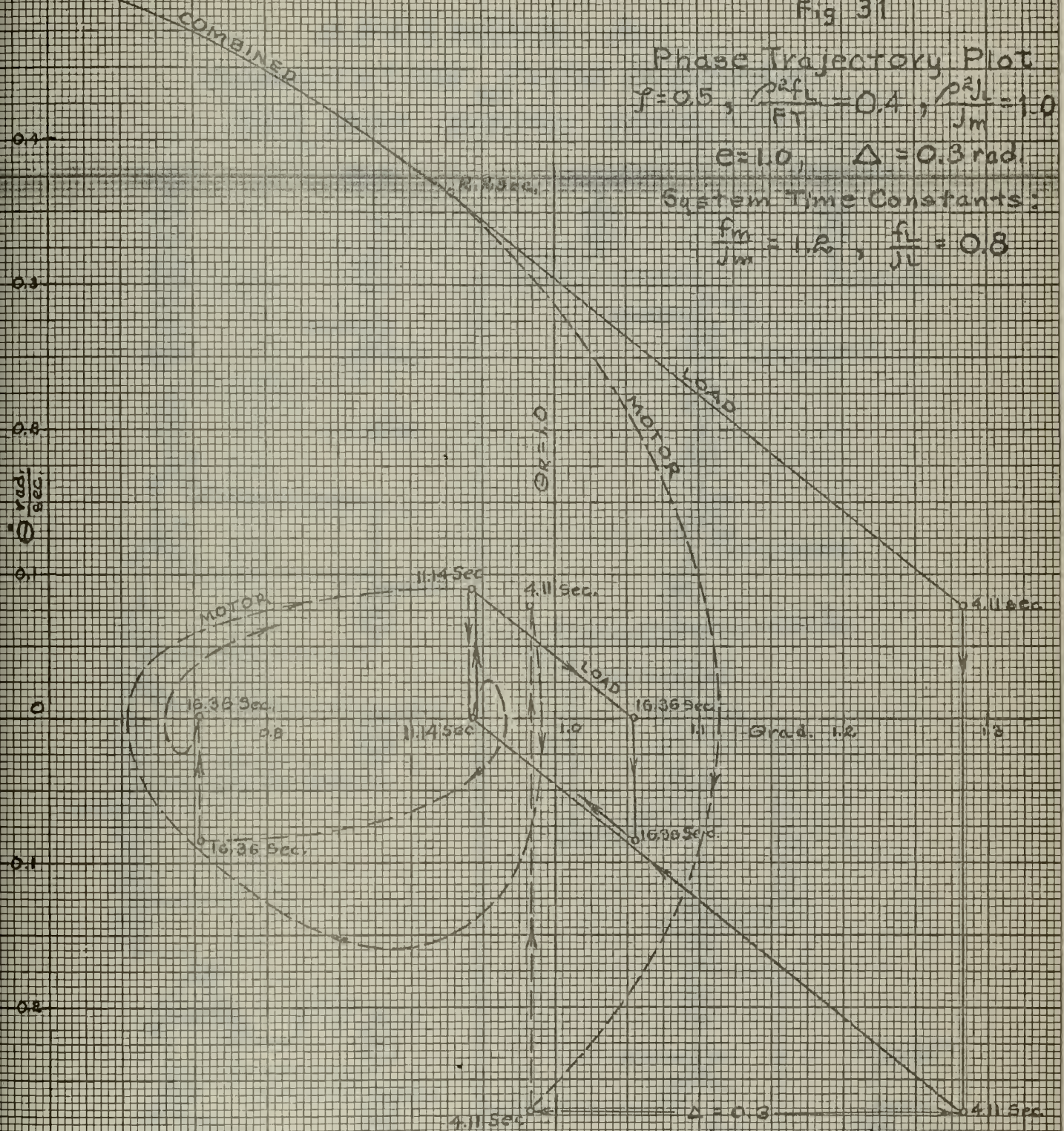
# Phase Trajectory Plot

$$\gamma = 0.5, \frac{p_{2f}}{F_T} = 0.4, \frac{p_{2j}}{J_m} = 1.0$$

$$e = 1.0, \Delta = 0.3 \text{ rad.}$$

System Time Constants:

$$\frac{f_m}{J_m} = 1.2, \frac{f_l}{J_l} = 0.8$$



Initial Conditions:

$$\theta_R = 1.0$$

$$\theta_c = \theta_m = 0$$

$$\dot{\theta}_c = \dot{\theta}_m = 0$$

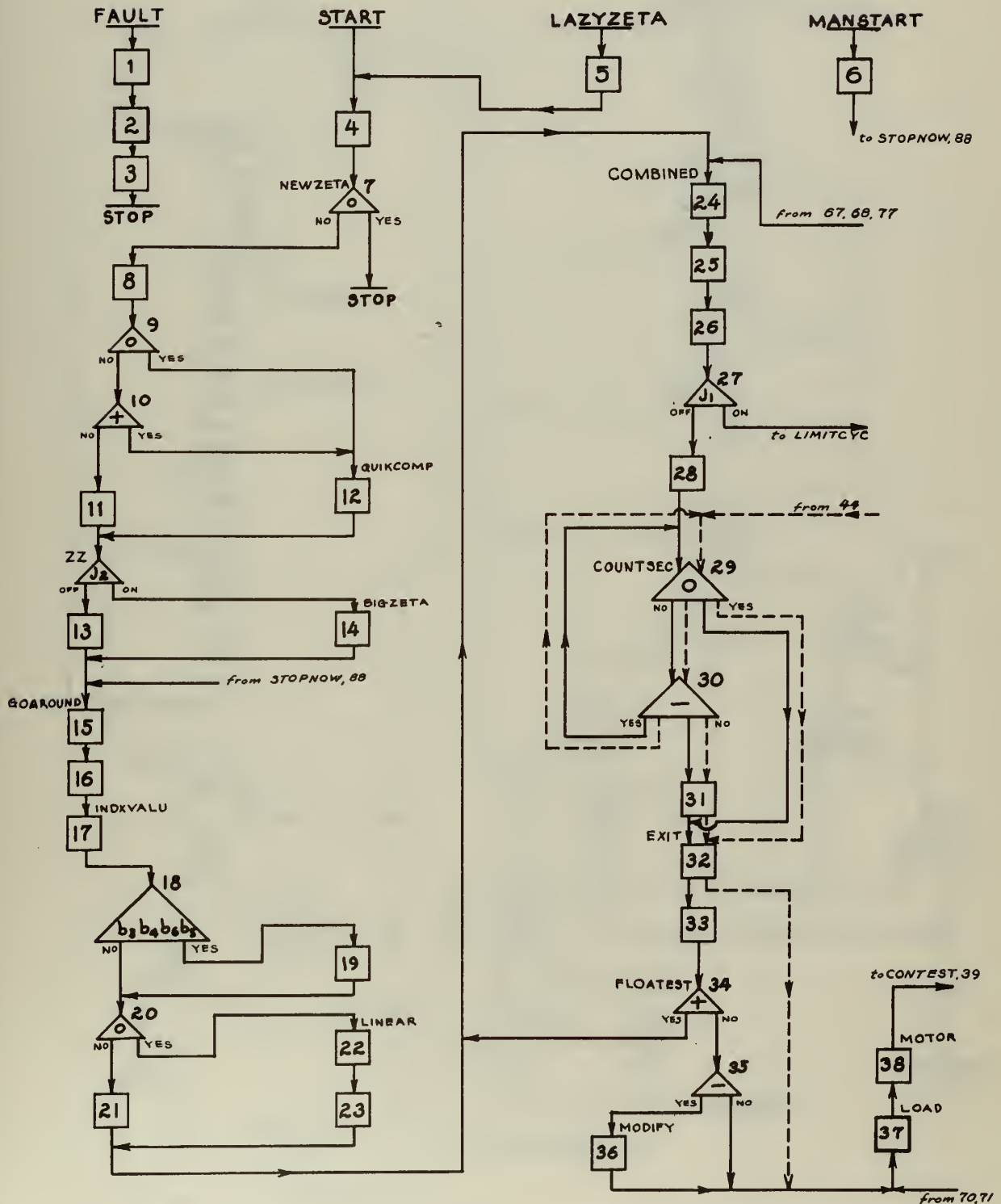
Steady State Error = 0.039 rad

See Figs 2 and 23

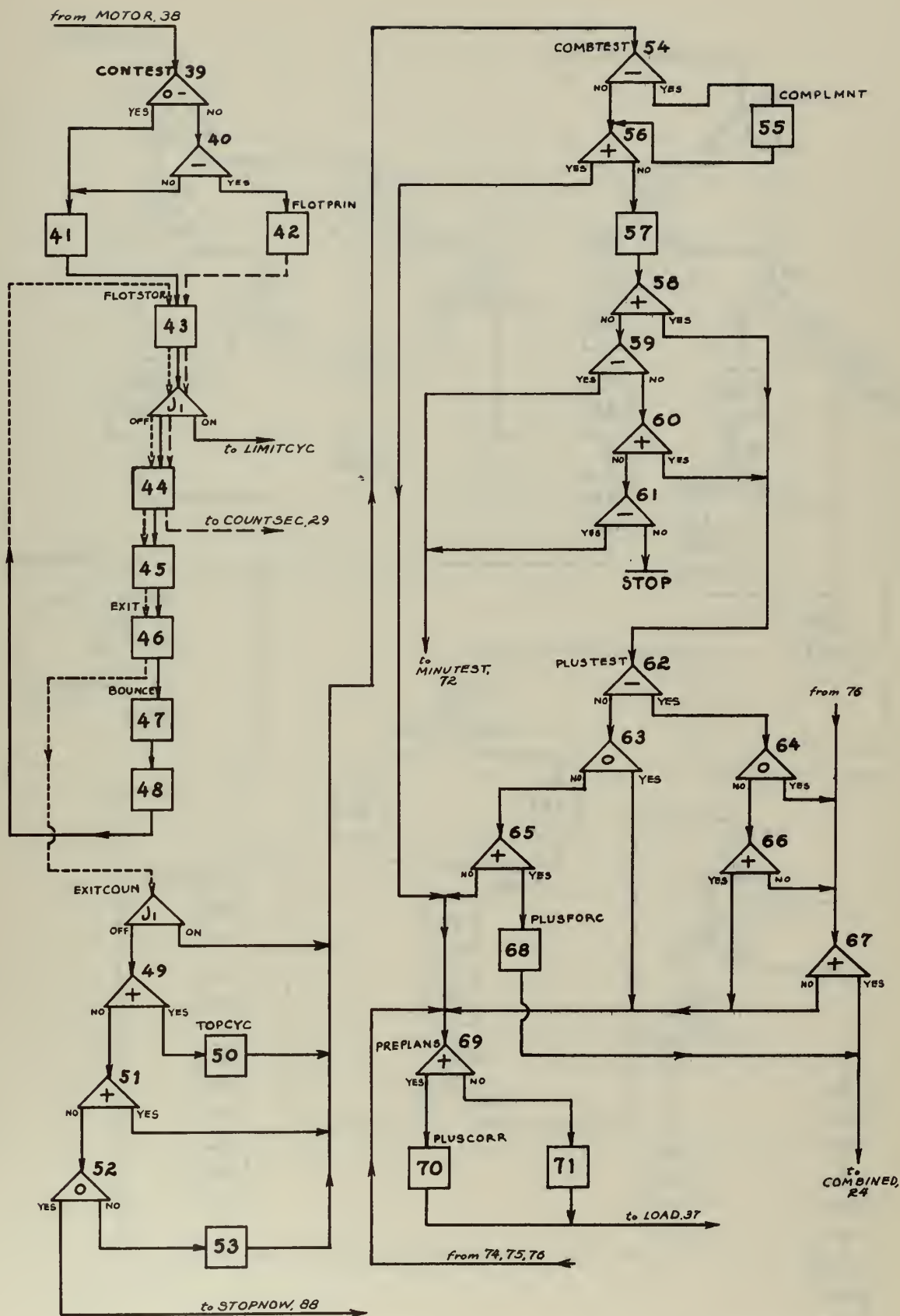


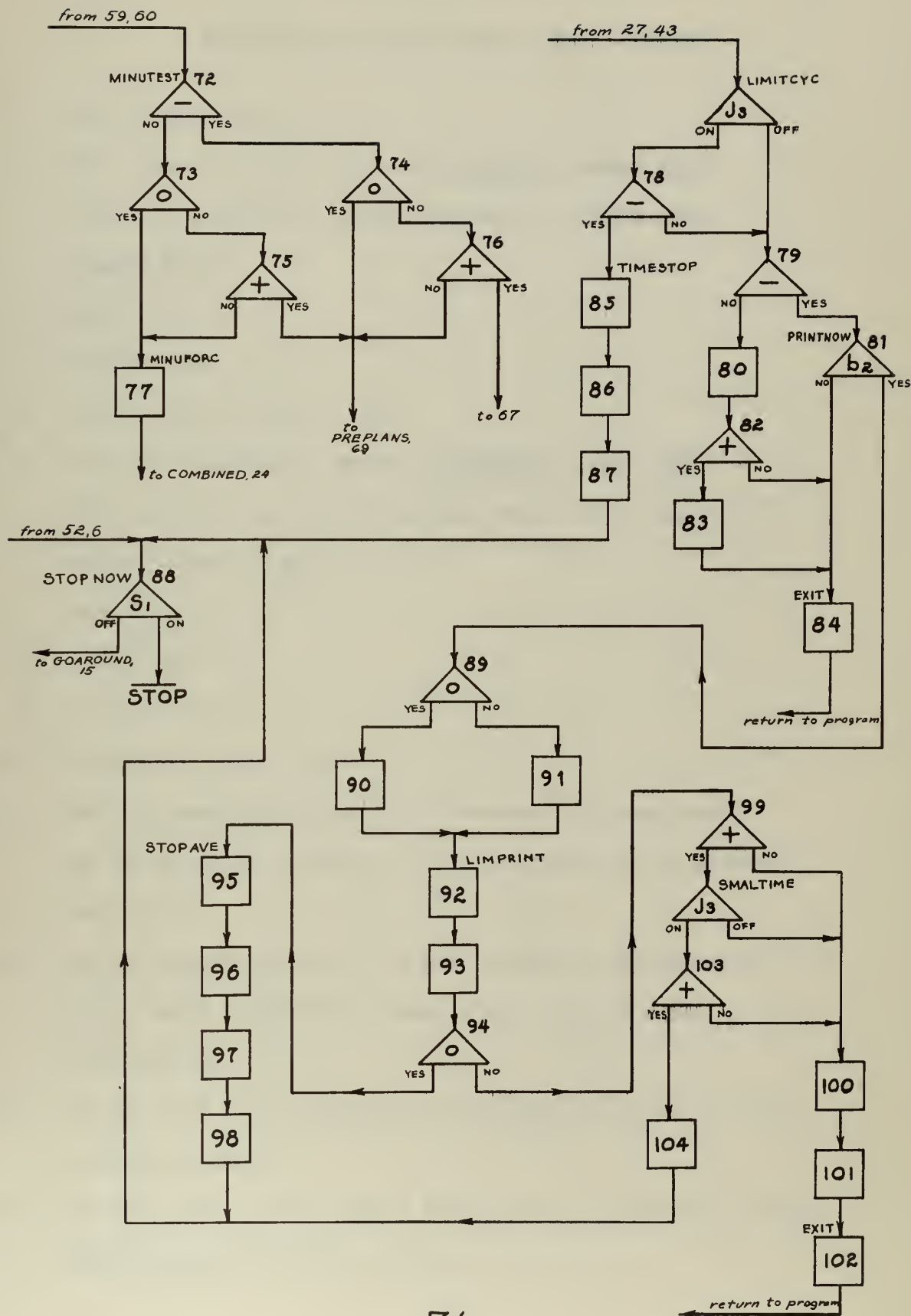
# APPENDIX A

## COMPUTATION FLOW DIAGRAM for CDC 1604 DIGITAL COMPUTER











APPENDIX A (continued)

EXPLANATION OF FLOW DIAGRAM BLOCK SYMBOLS

1. Clear fault stop.
2. Print last overshoot stored in LIMITBUF (symbol 888).  
Print out position in phase trajectory of fault stop  
(symbol 666).
3. End of file.
4. Actuate fault stop.
5. Load zeta from Index 1 ( $b_1$ ).
6. Print last overshoot stored on LIMITBUF (symbol 888).  
Print position in phase trajectory where it was manually  
stopped (symbol 1961).
7. Stop if Zeta = 0.
8.  $2 \int \omega_n = A$
9. Is zeta = 0.1?
10. Is zeta less than 0.1?
11. Set up computation time of 0.01 seconds for problem and  
set up for print at every 0.1 seconds when zeta is greater  
than 0.1.
12. Set up computation time of 0.004 seconds for problem and set  
up for print at every 0.1 seconds when zeta is equal to or  
less than 0.1.
13. Set up limit cycle print-out routine for 20 overshoots and  
average the last 8.
14. Set up number of limit cycle print-outs and averaging routine  
depending upon the size of zeta:

<u><math>\rho</math></u>	<u>Overshoots</u>	<u>Start Averaging</u>	<u>Divide by *</u>	<u>Comp. Time</u>
$\geq 0.8$	6	4	2.0	0.01
0.6	7	5	2.0	0.01
0.5	8	5	3.0	0.01
0.4	10	7	3.0	0.01
0.3	14	10	4.0	0.01
0.2	20	12	8.0	0.01
$\leq 0.1$	17	12	5.0	0.004

\*The last row printed has this number printed on the far left if the problem has a normal solution. This number indicates how many overshoots were averaged. The average value has 1.0 subtracted from it and is printed in the far right of the same last row.

15. Page reject.
16. Clear buffers and set up for new problem.
17. Restitution,  $e$ , from Index 3 ( $b_3$ ).  
Backlash,  $\Delta$ , from Index 4 ( $b_4$ ).  
 $J_m$  from Index 5 ( $b_5$ ).

$$\frac{J_T - J_m}{\rho^2} = J_L; \quad \frac{K}{J_m} = C; \quad \frac{J_m}{J_L}$$

$\frac{f_L}{F_T}$  from Index 6 ( $b_6$ ).

$$(f_L / F_T) (2 \rho \omega_n) (J_T) = f_L; \quad f_L / J_L = B$$

$$\frac{f_L}{f_L / F_T} - \rho^2 f_L = f_m; \quad f_m / J_m = D$$

18. Have all values been cycled through?
19. Set STOPNOW after this run to stop absolutely.



20. Backlash,  $\Delta$  , = 0?

21. Print consecutive number of this parameter run and all the parameters used:

$$\begin{array}{cccccc}
 2 \rho \omega_n = A & J_m/J_L & f_L/F_T & e & \Delta & \rho \\
 J_m & J_L & J_m + \rho^2 J_L & f_m & f_L & \rho^2 \\
 K & K/J_m = C & f_m/J_m = D & f_L/J_L = B & \omega_n^2 & \rho
 \end{array}$$

22. Set program so always acts as combined system.

23. Print consecutive number of this parameter run and selected parameters for linear system:

$$2 \rho \omega_n = A \quad J_m + \rho^2 J_L \quad K \quad \omega_n^2 \quad \rho \quad \rho^2$$

24. Solve the second order equation by Runge-Kutta-Gill numerical integration every 0.01 seconds problem time when zeta is greater than 0.1, otherwise every 0.004 seconds. Initial conditions are  $\Theta_R = 1.0$ ,  $\dot{\Theta}_c = \Theta_c = 0$  and  $\ddot{\Theta}_c = 1.0$  or the previously computed point. Next point is computed by four iterations of:

$$\ddot{\Theta}_c = \omega_n^2 (\Theta_R - \Theta_c) - 2 \rho \omega_n \dot{\Theta}_c$$

25. Set EXIT from print routine.

26. Load time,  $\dot{\Theta}_c$  and  $\Theta_c$  in print buffer.

27. Go to LIMITCYC to print only the overshoot. Otherwise print the phase trajectory points for each 0.1 seconds up to a maximum of 12 cycles.

28. Store 0 for  $\dot{\Theta}_m$  and  $\Theta_m$

29. Is problem time 0.1 sec.?

30. Is problem time greater than 0.1 sec.?

31. Print the number of point computed, time,  $\dot{\Theta}_c$  and  $\Theta_c$ .

32. EXIT (automatically set for desired jump-out.)

33. Store time,  $\ddot{\Theta}_c$ ,  $\dot{\Theta}_c$  and  $\Theta_c$  for motor and load initial conditions.

34. Is slope  $N_L = N_S$ ?

$$\frac{(\Theta_R - \Theta_c) \omega_n^2}{\dot{\Theta}_c} - 2\gamma \omega_n + \frac{f_L}{J_L} = \text{positive value?}$$

35. Is  $\dot{\Theta}_c$  negative?

36. Store  $\frac{\Theta_c - \Delta}{\rho}$  in  $\Theta_m$

37. Solve  $\ddot{\Theta}_c = -\left(\frac{f_L}{J_L}\right)(\dot{\Theta}_c)$  for  $\dot{\Theta}_c$  and  $\Theta_c$

by Runge-Kutta-Gill.

38. Solve  $\ddot{\Theta}_m = -\left(\frac{f_m}{J_m}\right)(\dot{\Theta}_m) + \left(\frac{K}{J_m}\right)(\Theta_R - \Theta_c)$

for  $\dot{\Theta}_m$  and  $\Theta_m$  by Runge-Kutta-Gill

39.  $\Theta_L - \rho \Theta_m \leq 0$ ?

40.  $\Theta_L - \rho \Theta_m - \Delta = \text{negative?}$

41. Set EXIT from print routine.

42. Set EXIT from print routine.

43. Load time,  $\dot{\Theta}_c$  and  $\Theta_c$ .

44. Load  $\dot{\Theta}_m$  and  $\Theta_m$ .

45. Print number of point computed and stored values.

46. EXIT (automatically set for desired jump-out).



47. Solve

$$\ddot{\Theta}'_c = \frac{J_m}{J_m + \rho^2 J_L} \left[ \rho \ddot{\Theta}_m (1+e) + \ddot{\Theta}_c \left( \rho^2 \frac{J_L}{J_m} - e \right) \right]$$

$$\ddot{\Theta}'_m = \frac{\ddot{\Theta}' - e(\rho \ddot{\Theta}_m + \ddot{\Theta}_c)}{\rho}$$

$$\ddot{\Theta}''_m = - \frac{f_m}{J_m} \ddot{\Theta}'_m + \frac{K}{J_m} (\Theta_R - \Theta_c)$$

48. Set EXIT from print routine.

49. Is  $\ddot{\Theta}_c$  positive?

50. Set up to count one phase trajectory when  $\ddot{\Theta}_c$  is negative.

51. Is  $\Theta_c > 1.0$ ?

52. Have 12 cycles of phase trajectory been completed?

53. Arrange for no counting until  $\ddot{\Theta}_c$  is positive again.

54. Is  $\rho \ddot{\Theta}_m - \ddot{\Theta}_c$  negative?

55. Set up  $-(\rho \ddot{\Theta}_m - \ddot{\Theta}_c)$ .

56. Is  $\rho \ddot{\Theta}_m - \ddot{\Theta}_c$  greater than preselected  $\epsilon$  ?

57. Set up combined system.

58. Is  $\ddot{\Theta}_c$  positive?

59. Is  $\ddot{\Theta}_c$  negative?

60. Is  $\Theta_c - 1$  positive?

61. Is  $\Theta_c - 1$  negative?

62. Is  $\Theta_c - 1$  negative?

63.  $\Theta_c - \rho \Theta_m = 0$ ?

64.  $\Theta_c - \rho \Theta_m = 0$ ?

65. Is  $\Theta_c - \rho \Theta_m$  positive?

66. Is  $\Theta_c - \rho \Theta_m$  positive?

67. Is Slope  $N_L = N_S$  ?

$$\frac{(1-\theta_c) \omega_n^2}{\dot{\theta}_c} - 2\rho \omega_n + \frac{f_L}{J_L} = \text{positive value?}$$

68. Modify FLOATEST to keep system combined. Takes off modification when  $\dot{\theta}_c$  is negative.

69. Is  $\theta_c - \rho \theta_m$  positive?

70. Store  $\frac{\theta_L - \Delta}{\rho}$  in  $\theta_m$ .

71. Store  $\frac{\theta_c}{\rho}$  in  $\theta_m$ .

72. Is  $\theta_c - 1$  negative?

73.  $\theta_c - \rho \theta_m = 0$  ?

74.  $\theta_c - \rho \theta_m = 0$  ?

75. Is  $\theta_c - \rho \theta_m$  positive?

76. Is  $\theta_c - \rho \theta_m$  positive?

77. Modify FLOATEST to keep system combined. Takes off modification when  $\dot{\theta}_c$  is positive.

78. Have 1 1/2 minutes of real time elapsed since the last print-out?

79. Is  $\theta_c < 1.0$ ?

80. Set up to print when in the fourth quadrant.

81. Check if in the fourth quadrant.

82. Is the new  $\theta_c$  greater than the previous  $\theta_c$ ?

83. Store the new time,  $\dot{\theta}_c$  and  $\theta_c$ .

84. EXIT (automatically set for desired jump-out).

85. Print the last overshoot stored in LIMITBUF (symbol 888)

86. Stop real time clock.

87. Print position in phase trajectory where it was stopped by the real time clock (symbol 999).



88. Return to beginning to start new problem if new parameters remain to be evaluated. Unconditional stop if all parameters have been evaluated.
89. Has the required number of overshoots been evaluated prior to starting the averaging routine?
90. Add the  $\Theta_c$  just evaluated to the previous sum for averaging.
91. Add one to the count prior to averaging.
92. Clear time clock.
93. Print time,  $\dot{\Theta}_c$  and  $\Theta_c$ .
94. Required number of total print-outs?
95. Clear buffers.
96. Obtain the average of  $\Theta_c$ .
97. Subtract 1.0.
98. Print the average of  $\Theta_c - 1.0$  and how many overshoots were utilized to obtain the average.
99. Is  $\Theta_c$  less than 1.005 radians?
100. Clear buffers.
101. Start real time clock.
102. EXIT (automatically set to desired jump-out).
103. Has 30 seconds of real time elapsed since the last print-out?
104. Print the position in the phase trajectory where it was stopped by the real time clock and  $\Theta_c$  being less than 1.005 radians (symbol 1005).

#### Jump Switches:

$J_1$  - For LIMITCYC, allows the maximum overshoot print-out only.

With  $J_1$  down, a phase trajectory is printed for a maximum of 12 cycles.

$J_2$  - Set the number of print-outs depending on zeta. Change the number of overshoot values used to obtain the average value.

$J_3$  - Use the real time clock for an automatic recycle to a new parameter when have:

- a. 1 1/2 minutes maximum after the last print-out.
- b. 30 seconds maximum and  $\theta_c$  is 1.005 radians or less.

#### Stop Switches:

$S_1$  - Stops at the end of this parameter run or stops at the end of the manual print-out.



## APPENDIX B

## COMPUTER PROGRAM

00007	74 0 00070	FAULT	ORG 00007
	75 0 40701		EXF 0 00070
			SLJ 0 FAULPRIN
40000	74 0 00100	START	ORG 40000
	75 0 40004		EXF 0 00100
40001	12 1 40623	LAZY ZETA	SLJ 0 NEWZETA-1
	20 0 40425		LDA 1 ZETAINDX
40002	50 1 00000		STA 0 ZETA
	75 0 40000		ENI 1 0
40003	75 0 40674	MANSTART	SLJ 0 START
	50 0 00000		SLJ 0 MANUAL
40004	12 0 40565		ENI 0 0
	20 0 40563		LDA 0 NEWSTOP
40005	12 0 40425	NEWZETA	STA 0 STOPNOW
	22 0 40564		LDA 0 ZETA
40006	32 0 40665		AJP 0 STOPINDX
	32 0 40426		FMU 0 TWO
40007	20 0 40404		FMU 0 OMEGAN
	12 0 40624		STA 0 A
40010	31 0 40425		LDA 0 ZETAINDX+1
	22 0 40433		FSB 0 ZETA
40011	22 2 40433		AJP 0 QUIKCOMP
	12 0 40402		AJP 2 QUIKCOMP
40012	20 0 40154		LDA 0 POINT01
	20 0 40173		STA 0 TABLES+1
40013	20 0 40207		STA 0 TABLEL+1
	12 0 40661		STA 0 TABLEM+1
40014	20 0 40400		LDA 0 COUNT10
	50 0 00000		STA 0 TENTHSEC
40015	75 2 40437	ZZ	ENI 0 0
	50 0 00000		SLJ 2 BIGZETA
40016	12 0 40652		ENI 0 0
	20 0 40616		LDA 0 STOP20
40017	12 0 40662		STA 0 STOPZETA
	20 0 40617		LDA 0 COUNT12
40020	12 0 40672		STA 0 COUNTZET
	20 0 40620		LDA 0 EIGHT
40021	10 0 00010		STA 0 DIVIDE
	61 0 40562		ENA 0 10
40022	75 4 71000	GOAROUND	SAL 0 STOPPRINT
	50 0 00000		SLJ 4 DECOF
40023	02 0 40427		ENI 0 0
	04 0 00000		02 0 FORZEROS
40024	12 0 40664		04 0 0
	20 0 40156		LDA 0 ONE
40025	10 0 00000		STA 0 UDOT
	20 0 00000		ENA 0 0
40026	20 0 40162		STA 0 0
	20 0 40161		STA 0 THETA
40027	20 0 40157		STA 0 THETADOT
	20 0 40155		STA 0 U
40030	20 0 40377		STA 0 T
	20 0 40615		STA 0 INDEX
40031	20 0 40351		STA 0 INDEXAVE
	20 0 40352		STA 0 PRINTBUF+3
40032	20 0 40353		STA 0 PRINTBUF+4
	20 0 40614		STA 0 PRINTBUF+5
40033	20 0 40607		STA 0 LIMINDEX
	20 0 40610		STA 0 LIMITBUF+2
40034	20 0 40611		STA 0 LIMITBUF+3
	20 0 40612		STA 0 LIMITBUF+4
40035	20 0 40613		STA 0 LIMITBUF+5
	12 0 40621		STA 0 LIMITBUF+6
40036	20 0 40326		LDA 0 PRINTSET
	20 0 40335		STA 0 OK2PRINT+1
40037	12 0 40622		STA 0 BOUNPRIN+4
	20 0 40544		LDA 0 NEWLIMPT
40040	12 0 40253		STA 0 LIMPRINT+1
	20 0 40071		LDA 0 FLOTDATA
40041	12 0 40254		STA 0 FLOATEST
	20 0 40072		LDA 0 FLOTDATA+1
40042	75 0 40452	NEWINPUT	STA 0 FLOATEST+1
	50 0 00000		SLJ 0 INDXVALU
			ENI 0 00000

40043	12	0	40410	LDA	0	DELTA
	22	0	40500	AJP	0	LINEAR
40044	12	0	40513	LDA	0	NOSINK
	20	0	40071	STA	0	FLOATEST
40045	75	4	71000	SLJ	4	DECOF
	50	0	00000	ENI	0	0
40046	01	0	40404	01	0	A
	06	0	00001	06	0	1
40047	72	0	40046	RAO	0	7-1
	75	4	71000	SLJ	4	DECOF
40050	01	0	40412	01	0	MOINERT
	06	0	00000	06	0	0
40051	72	0	40510	RAO	0	LINEAR+10
	75	4	71000	SLJ	4	DECOF
40052	01	0	40420	01	0	KMOTCONS
	06	0	00000	06	0	0
40053	10	0	00000	COMBINED ENA	0	0
	20	0	40160	STA	0	QX
40054	20	0	40163	STA	0	QY
	75	4	60200	SLJ	4	RUNGE
40055	00	0	40153	0	0	TABLES
	00	0	40164	0	0	DERIVES
40056	75	4	60201	SLJ	4	RUNGE+1
	50	0	00000	ENI	0	0
40057	75	0	40310	SLJ	0	COMBPRIN
	50	0	00000	ENI	0	0
40060	12	0	40155	LDA	0	T
	20	0	40174	STA	0	TL
40061	20	0	40210	STA	0	TM
	12	0	40156	LDA	0	UDOT
40062	20	0	40175	STA	0	VDOT
	33	0	40411	FDV	0	RHO
40063	20	0	40211	STA	0	WDOT
	12	0	40157	LDA	0	U
40064	20	0	40176	STA	0	V
	20	0	40200	STA	0	THETADL
40065	33	0	40411	FDV	0	RHO
	20	0	40212	STA	0	W
40066	20	0	40214	STA	0	THETADM
	12	0	40162	LDA	0	THETA
40067	20	0	40201	STA	0	THETAL
	33	0	40411	FDV	0	RHO
40070	20	0	40215	STA	0	THETAM
	50	0	00000	ENI	0	0
40071	13	0	40162	FLOATEST LAC	0	THETA
	30	0	40664	FAD	0	ONE
40072	32	0	40424	FMU	0	OMEGANSQ
	33	0	40161	FDV	0	THETADOT
40073	31	0	40404	FSB	0	A
	30	0	40423	FAD	0	B
40074	22	2	40053	AJP	2	COMBINED
	12	0	40161	LDA	0	THETADOT
40075	22	3	40270	AJP	3	MODIFY
	50	0	00000	ENI	0	0
40076	10	0	00000	LOAD ENA	0	0
	20	0	40177	STA	0	QL
40077	20	0	40202	STA	0	QLL
	20	0	40213	STA	0	QM
40100	20	0	40216	STA	0	QMM
	75	4	60200	SLJ	4	RUNGE
40101	00	0	40172	0	0	TABLEL
	00	0	40203	0	0	DERIVL
40102	75	4	60201	SLJ	4	RUNGE+1
	50	0	00000	ENI	0	0
40103	75	4	60200	MOTOR SLJ	4	RUNGE
	50	0	00000	ENI	0	0
40104	00	0	40206	0	0	TABLEM
	00	0	40217	0	0	DERIVM
40105	75	4	60201	SLJ	4	RUNGE+1
	50	0	00000	ENI	0	0
40106	13	0	40215	CONTEST LAC	0	THETAM
	32	0	40411	FMU	0	RHO
40107	30	0	40201	FAD	0	THETAL
	22	0	40112	AJP	0	BOUNCE



40110	22	3	40112
	31	0	40410
40111	22	3	40317
	50	0	00000
40112	75	0	40331
	50	0	00000
40113	12	0	40200
	20	0	40212
40114	12	0	40417
	33	0	40405
40115	31	0	40407
	32	0	40200
40116	20	0	40200
	12	0	40664
40117	30	0	40407
	32	0	40411
40120	32	0	40214
	30	0	40200
40121	33	0	40414
	32	0	40412
40122	20	0	40200
	20	0	40176
40123	13	0	40176
	32	0	40423
40124	20	0	40175
	13	0	40214
40125	32	0	40411
	30	0	40212
40126	32	0	40407
	30	0	40200
40127	33	0	40411
	20	0	40214
40130	20	0	40212
	13	0	40212
40131	32	0	40422
	20	0	40211
40132	13	0	40201
	30	0	40664
40133	32	0	40421
	30	0	40211
40134	20	0	40211
	75	0	40332
40135	50	0	00000
	12	0	40214
40136	32	0	40411
	31	0	40200
40137	22	3	40276
	50	0	00000
40140	65	0	40403
	75	0	40301
40141	12	0	40200
	20	0	40161
40142	20	0	40157
	13	0	40157
40143	32	0	40404
	20	0	40156
40144	12	0	40201
	20	0	40162
40145	13	0	40162
	30	0	40664
40146	32	0	40424
	30	0	40156
40147	20	0	40156
	12	0	40174
40150	20	0	40155
	12	0	40200
40151	22	2	40225
	22	3	40255
40152	75	0	40273
	50	0	00000
40153	00	0	00000
	00	0	00002
40154	17	7	15075
	34	1	21727

# BOUNCE

# COMBTES

# TABLES

AJP	3	BOUNCE
FSB	0	DELTA
AJP	3	FLOTPRIN
ENI	0	0
SLJ	0	BOUNPRIN
ENI	0	0
LDA	0	THETADL
STA	0	W
LDA	0	RHOSQ
FDV	0	INERTRAT
FSB	0	RESTITUT
FMU	0	THETADL
STA	0	THETADL
LDA	0	ONE
FAD	0	RESTITUT
FMU	0	RHO
FMU	0	THETADM
FAD	0	THETADL
FDV	0	TOTINERT
FMU	0	MOINERT
STA	0	THETADL
STA	0	V
LAC	0	V
FMU	0	B
STA	0	VDOT
LAC	0	THETADM
FMU	0	RHO
FAD	0	W
FMU	0	RESTITUT
FAD	0	THETADL
FDV	0	RHO
STA	0	THETADM
STA	0	W
LAC	0	W
FMU	0	D
STA	0	WDOT
LAC	0	THETAL
FAD	0	ONE
FMU	0	C
FAD	0	WDOT
STA	0	WDOT
SLJ	0	BOUNPRIN+1
ENI	0	0
LDA	0	THETADM
FMU	0	RHO
FSB	0	THETADL
AJP	3	COMPLMNT
ENI	0	00000
THS	0	DIFFERNT
SLJ	0	PREPLANS
LDA	0	THETADL
STA	0	THETADOT
STA	0	U
LAC	0	U
FMU	0	A
STA	0	UDOT
LDA	0	THETAL
STA	0	THETA
LAC	0	THETA
FAD	0	ONE
FMU	0	OMEGANSQ
FAD	0	UDOT
STA	0	UDOT
LDA	0	TL
STA	0	T
LDA	0	THETADL
AJP	2	PLUSTEST
AJP	3	MINUTEST
SLJ	0	TESTZERO
ENI	0	0
OCT	2	
DEC	.01	

40155	00 0 00000	T	DEC	0
40156	00 0 00000	UDOT	DEC	1.0
40157	00 0 00000	U	DEC	0
40160	00 0 00000	QX	DEC	0
40161	00 0 00000	THETADOT	DEC	0
40162	00 0 00000	THETA	DEC	0
40163	00 0 00000	QY	DEC	0
40164	13 0 40157	DERIVES	LAC	0 U
	32 0 40404		FMU	0 A
40165	20 0 40156		STA	0 UDOT
	13 0 40162		LAC	0 THETA
40166	30 0 40664		FAD	0 ONE
	32 0 40424		FMU	0 OMEGANSQ
40167	30 0 40156		FAD	0 UDOT
	20 0 40156		STA	0 UDOT
40170	12 0 40157		LDA	0 U
	20 0 40161		STA	0 THETADOT
40171	75 0 60202		SLJ	0 RUNGE+2
	50 0 00000		ENI	0 0
40172	00 0 00000	TABLEL	OCT	2
	00 0 00002			
40173	17 7 15075		DEC	.01
	34 1 21727			
40174	00 0 00000	TL	DEC	0
	00 0 00000			
40175	00 0 00000	VDOT	DEC	0
	00 0 00000			
40176	00 0 00000	V	DEC	0
	00 0 00000			
40177	00 0 00000	QL	DEC	0
	00 0 00000			
40200	00 0 00000	THETADL	DEC	0
	00 0 00000			
40201	00 0 00000	THETAL	DEC	0
	00 0 00000			
40202	00 0 00000	QLL	DEC	0
	00 0 00000			
40203	13 0 40176	DERIVL	LAC	0 V
	32 0 40423		FMU	0 B
40204	20 0 40175		STA	0 VDOT
	12 0 40176		LDA	0 V
40205	20 0 40200		STA	0 THETADL
	75 0 60202		SLJ	0 RUNGE+2
40206	00 0 00000	TABLEM	OCT	2
	00 0 00002			
40207	17 7 15075		DEC	.01
	34 1 21727			
40210	00 0 00000	TM	DEC	0
	00 0 00000			
40211	00 0 00000	WDOT	DEC	0
	00 0 00000			
40212	00 0 00000	W	DEC	0
	00 0 00000			
40213	00 0 00000	QM	DEC	0
	00 0 00000			
40214	00 0 00000	THETADM	DEC	0
	00 0 00000			
40215	00 0 00000	THETAM	DEC	0
	00 0 00000			
40216	00 0 00000	QMM	DEC	0
	00 0 00000			
40217	13 0 40201	DERIVM	LAC	0 THETAL
	32 0 40421		FMU	0 C
40220	20 0 40211		STA	0 WDOT
	13 0 40212		LAC	0 W
40221	32 0 40422		FMU	0 D
	30 0 40421		FAD	0 C



40222	30	0	40211	FAD	0	WDOT
	20	0	40211	STA	0	WDOT
40223	12	0	40212	LDA	0	W
	20	0	40214	STA	0	THETADM
40224	75	0	60202	SLJ	0	RUNGE+2
	50	0	00000	ENI	0	0
40225	12	0	40201	PLUSTEST LDA	0	THETAL
	31	0	40664	FSB	0	ONE
40226	22	3	40232	AJP	3	PLUSTEST+5
	13	0	40215	LAC	0	THETAM
40227	32	0	40411	FMU	0	RHO
	30	0	40201	FAD	0	THETAL
40230	22	0	40301	AJP	0	PREPLANS
	22	2	40241	AJP	2	PLUSFORC
40231	75	0	40301	SLJ	0	PREPLANS
	50	0	00000	ENI	0	0
40232	13	0	40215	LAC	0	THETAM
	32	0	40411	FMU	0	RHO
40233	30	0	40201	FAD	0	THETAL
	22	0	40235	AJP	0	PLUSTEST+10
40234	22	2	40301	AJP	2	PREPLANS
	50	0	00000	ENI	0	0
40235	13	0	40162	LAC	0	THETA
	30	0	40664	FAD	0	ONE
40236	32	0	40424	FMU	0	OMEGANSQ
	33	0	40161	FDV	0	THETADOT
40237	31	0	40404	FSB	0	A
	30	0	40423	FAD	0	B
40240	22	2	40053	AJP	2	COMBINED
	75	0	40301	SLJ	0	PREPLANS
40241	12	0	40244	PLUSFORC LDA	0	NUMB
	20	0	40071	STA	0	FLOATEST
40242	12	0	40245	LDA	0	NUMB+1
	20	0	40072	STA	0	FLOATEST+1
40243	75	0	40053	SLJ	0	COMBINED
	50	0	00000	ENI	0	0
40244	12	0	40161	NUMB LDA	0	THETADOT
	22	2	40053	AJP	2	COMBINED
40245	75	0	40250	SLJ	0	RESTORET
	50	0	00000	ENI	0	0
40246	12	0	40161	DUMB LDA	0	THETADOT
	22	3	40053	AJP	3	COMBINED
40247	75	0	40250	SLJ	0	RESTORET
	50	0	00000	ENI	0	0
40250	12	0	40253	RESTORET LDA	0	FLOTDATA
	20	0	40071	STA	0	FLOATEST
40251	12	0	40254	LDA	0	FLOTDATA+1
	20	0	40072	STA	0	FLOATEST+1
40252	75	0	40053	SLJ	0	COMBINED
	50	0	00000	ENI	0	0
40253	13	0	40162	FLOTDATA LAC	0	THETA
	30	0	40664	FAD	0	ONE
40254	32	0	40424	FMU	0	OMEGANSQ
	33	0	40161	FDV	0	THETADOT
40255	12	0	40201	MINUTEST LDA	0	THETAL
	31	0	40664	FSB	0	ONE
40256	22	2	40262	AJP	2	MINUTEST+5
	13	0	40215	LAC	0	THETAM
40257	32	0	40411	FMU	0	RHO
	30	0	40201	FAD	0	THETAL
40260	22	0	40265	AJP	0	MINUFORC
	22	2	40301	AJP	2	PREPLANS
40261	75	0	40265	SLJ	0	MINUFORC
	50	0	00000	ENI	0	0
40262	13	0	40215	LAC	0	THETAM
	32	0	40411	FMU	0	RHO
40263	30	0	40201	FAD	0	THETAL
	22	0	40301	AJP	0	PREPLANS
40264	22	2	40235	AJP	2	PLUSTEST+10
	75	0	40301	SLJ	0	PREPLANS
40265	12	0	40246	MINUFORC LDA	0	DUMB
	20	0	40071	STA	0	FLOATEST
40266	12	0	40247	LDA	0	DUMB+1
	20	0	40072	STA	0	FLOATEST+1

40267	75	0	40053		SLJ	0	COMBINED
	50	0	00000		ENI	0	0
40270	12	0	40162	MODIFY	LDA	0	THETA
	31	0	40410		FSB	0	DELTA
40271	33	0	40411		FDV	0	RHO
	20	0	40215		STA	0	THETAM
40272	75	0	40076		SLJ	0	LOAD
	50	0	00000		ENI	0	0
40273	12	0	40201	TESTZERO	LDA	0	THETAL
	31	0	40664		FSB	0	ONE
40274	22	2	40225		AJP	2	PLUSTEST
	22	3	40255		AJP	3	MINUTEST
40275	76	0	00000		SLS	0	0
	50	0	00000		ENI	0	0
40276	20	0	40300	COMPLMNT	STA	0	COMPLMNT+2
	13	0	40300		LAC	0	COMPLMNT+2
40277	75	0	40140		SLJ	0	COMBTST+3
	50	0	00000		ENI	0	0
40300	00	0	00000		BSS	1	
	00	0	00000				
40301	13	0	40215	PREPLANS	LAC	0	THETAM
	32	0	40411		FMU	0	RHO
40302	30	0	40201		FAD	0	THETAL
	22	2	40305		AJP	2	PLUSCORR
40303	12	0	40201		LDA	0	THETAL
	33	0	40411		FDV	0	RHO
40304	20	0	40215		STA	0	THETAM
	75	0	40076		SLJ	0	LOAD
40305	12	0	40201	PLUSCORR	LDA	0	THETAL
	31	0	40410		FSB	0	DELTA
40306	33	0	40411		FDV	0	RHO
	20	0	40215		STA	0	THETAM
40307	75	0	40076		SLJ	0	LOAD
	50	0	00000		ENI	0	0
40310	12	0	40354	COMBPRIN	LDA	0	COMBEXIT
	20	0	40360		STA	0	EXIT
40311	12	0	40155		LDA	0	T
	20	0	40346		STA	0	PRINTBUF
40312	12	0	40157		LDA	0	U
	20	0	40347		STA	0	PRINTBUF+1
40313	12	0	40162		LDA	0	THETA
	20	0	40350		STA	0	PRINTBUF+2
40314	75	1	40522		SLJ	1	LIMITCYC
	10	0	00000		ENA	0	0
40315	20	0	40351		STA	0	PRINTBUF+3
	20	0	40352		STA	0	PRINTBUF+4
40316	75	0	40321		SLJ	0	COUNTSEC
	50	0	00000		ENI	0	0
40317	12	0	40355	FLOTPRIN	LDA	0	FLOTEXIT
	20	0	40360		STA	0	EXIT
40320	75	4	40337		SLJ	4	FLOTSTOR
	50	0	00000		ENI	0	0
40321	72	0	40377	COUNTSEC	RAO	0	INDEX
	12	0	40377		LDA	0	INDEX
40322	15	0	40400		SUB	0	TENTHSEC
	22	0	40325		AJP	0	OK2PRINT
40323	22	2	40322		AJP	2	COUNTSEC+1
	14	0	40400		ADD	0	TENTHSEC
40324	20	0	40377		STA	0	INDEX
	75	0	40360		SLJ	0	EXIT
40325	75	4	71000	OK2PRINT	SLJ	4	DECOF
	50	0	00000		ENI	0	0
40326	00	0	40346		00	0	PRINTBUF
	06	0	00001		06	0	1
40327	72	0	40326		RAO	0	/-1
	10	0	00000		ENA	0	0
40330	20	0	40377		STA	0	INDEX
	75	0	40360		SLJ	0	EXIT
40331	12	0	40356	BOUNPRIN	LDA	0	BOUNEXIT
	75	0	40333		SLJ	0	BOUNPRIN+2
40332	12	0	40357		LDA	0	BOUNEXIT+1
	50	0	00000		ENI	0	0
40333	20	0	40360		STA	0	EXIT
	75	4	40337		SLJ	4	FLOTSTOR



40334	75 4	71000	SLJ	4	DECOF
	50 0	00000	ENI	0	0
40335	00 0	40346		0	PRINTBUF
	06 0	00001		0	1
40336	75 0	40360	SLJ	0	EXIT
	50 0	00000	ENI	0	0
40337	75 0	00000	SLJ	0	0
	12 0	40174	LDA	0	TL
40340	20 0	40346	STA	0	PRINTBUF
	12 0	40176	LDA	0	V
40341	20 0	40347	STA	0	PRINTBUF+1
	12 0	40201	LDA	0	THETAL
40342	20 0	40350	STA	0	PRINTBUF+2
	75 1	40322	SLJ	1	LIMITCYC
40343	12 0	40212	LDA	0	W
	20 0	40351	STA	0	PRINTBUF+3
40344	12 0	40215	LDA	0	THETAM
	20 0	40352	STA	0	PRINTBUF+4
40345	75 0	40337	SLJ	0	FLOTSTOR
	50 0	00000	ENI	0	0
40346	00 0	00000	PRINTBUF	BSS	6
	00 0	00000			
40354	75 0	40060	COMBEXIT	SLJ	0 COMBINED+5
	50 0	00000	ENI	0	0
40355	75 0	40076	FLOTEXIT	SLJ	0 LOAD
	50 0	00000	ENI	0	0
40356	75 0	40113	BOUNEXIT	SLJ	0 BOUNCE+1
	50 0	00000	ENI	0	0
40357	75 0	40361		SLJ	0 EXITCOUN
	50 0	00000	ENI	0	0
40360	00 0	00000	EXIT	BSS	1
	00 0	00000			
40361	75 1	40135	EXITCOUN	SLJ	1 COMBTTEST
	72 0	40377	RAO	0	INDEX
40362	72 0	40335	RAO	0	BOUNPRIN+4
	12 0	40347	LDA	0	PRINTBUF+1
40363	22 2	40372	AJP	2	TOPCYC
	12 0	40350	LDA	0	PRINTBUF+2
40364	31 0	40664	FSB	0	ONE
	22 2	40135	AJP	2	COMBTTEST
40365	72 0	40615	RAO	0	INDEXAVE
	12 0	40615	LDA	0	INDEXAVE
40366	15 0	40662	SUB	0	COUNT12
	22 0	40563	AJP	0	STOPNOW
40367	14 0	40662	ADD	0	COUNT12
	20 0	40615	STA	0	INDEXAVE
40370	12 0	40375	LDA	0	NEGAJUMP
	20 0	40364	STA	0	EXITCOUN+3
40371	75 0	40135	SLJ	0	COMBTTEST
	50 0	00000	ENI	0	00000
40372	12 0	40350	TOPCYC	LDA	0 PRINTBUF+2
	31 0	40664	FSB	0	ONE
40373	22 3	40135	AJP	3	COMBTTEST
	12 0	40376	LDA	0	NEWLOWER
40374	20 0	40364	STA	0	EXITCOUN+3
	75 0	40135	SLJ	0	COMBTTEST
40375	75 0	40135	NEGAJUMP	SLJ	0 COMBTTEST
	50 0	00000	ENI	0	0
40376	31 0	40664	NEWLOWER	FSB	0 ONE
	22 2	40135	AJP	2	COMBTTEST
40377	00 0	00000	INDEX	OCT	0
	00 0	00000			
40400	00 0	00000	TENTHSEC	OCT	12
	00 0	00012			
40401	17 7	04061	POINT004	DEC	0.004
	11 5	64570			
40402	17 7	15075	POINT01	DEC	0.01
	34 1	21727			
40403	17 3	74000	DIFFERNT	OCT	173740000000000000
	00 0	00000			
40404	20 0	06314	A	DEC	.8
	63 1	46314			
40405	20 0	14000	INERTRAT	DEC	1.0
	00 0	00000			

40406	00 0 00000	FLFTPRIN BSS	1
40407	00 0 00000	RESTITUT DEC	0
40410	17 7 64631	DELTA DEC	.3
40411	46 3 14631	RHO DEC	1.0
40412	20 0 14000	MOINERT DEC	.5
40413	00 0 00000	JLOADVAL BSS	1
40414	00 0 00000	TOTINERT DEC	1.0
40415	20 0 14000	FMOTOR BSS	1
40416	00 0 00000	FLOAD BSS	1
40417	00 0 00000	RHOSQ DEC	1.0
40420	20 0 14000	KMOTCONS DEC	1.0
40421	00 0 00000	C DEC	2.0
40422	20 0 24000	D DEC	.4
40423	17 7 66314	B DEC	.01
40424	63 1 46314	OMEGANSQ DEC	1.0
40425	17 7 15075	ZETA BSS	1
40426	34 1 21727	OMEGAN DEC	1.0
40427	20 0 14000	FORZEROS BSS	4
	00 0 00000		
40433	12 0 40401	RUNGE EQU	60200
40434	20 0 40154	DECOF EQU	71000
40435	20 0 40173	QUICKCOMP LDA	0 POINT004
40436	20 0 40207	STA	0 TABLES+1
40437	12 0 40654	STA	0 TABLEL+1
40438	20 0 40400	STA	0 TABLEM+1
40439	75 0 40015	LDA	0 STOP25
40440	50 0 00000	STA	0 TENTHSEC
40441	50 1 00000	SLJ	0 ZZ
40442	50 0 00000	ENI	0 0
40443	12 0 40425	BIGZETA ENI	1 0
40444	31 1 40636	ENI	0 0
40445	22 2 40444	LDA	0 ZETA
40446	22 0 40444	FSB	1 POINT8
40447	54 1 00006	AJP	2 ZETAPLUS
40448	75 0 40440	AJP	0 ZETAPLUS
40449	50 1 00006	ISK	1 6
40450	50 0 00000	SLJ	0 /-2
40451	12 1 40645	ENI	1 6
40452	20 0 40616	ENI	0 0
40453	12 1 40655	ZETAPLUS LDA	1 STOP6
40454	20 0 40617	STA	0 STOPZETA
40455	12 1 40665	LDA	1 COUNT4
40456	20 0 40620	STA	0 COUNTZET
40457	12 1 40645	LDA	1 TWO
40458	15 1 40655	STA	0 DIVIDE
40459	61 0 40562	LDA	1 STOP6
40460	50 1 00000	SUB	1 COUNT4
40461	75 0 40022	SAL	0 STOPPRINT
40462	50 0 00000	ENI	1 0
40463	12 3 40706	SLJ	0 GOAROUND
40464	20 0 40407	ENI	0 0
40465	12 4 40726	INDXVALU LDA	3 RESTVALU
40466	20 0 40410	STA	0 RESTITUT
40467	12 5 40747	LDA	4 DELTAVAL
40468	20 0 40412	STA	0 DELTA
40469		LDA	5 JMOTVALU
40470		STA	0 MOINERT



40455	12	0	40414	LDA	0	TOTINERT
	31	0	40412	FSB	0	MOINERT
40456	33	0	40417	FDV	0	RHOSQ
	20	0	40413	STA	0	JLOADVAL
40457	12	0	40420	LDA	0	KMOTCONS
	33	0	40412	FDV	0	MOINERT
40460	20	0	40421	STA	0	C
	12	0	40412	LDA	0	MOINERT
40461	33	0	40413	FDV	0	JLOADVAL
	20	0	40405	STA	0	INERTRAT
40462	12	6	40774	LDA	6	FLFTRATO
	20	0	40406	STA	0	FLFTPRIN
40463	32	0	40404	FMU	0	A
	32	0	40414	FMU	0	TOTINERT
40464	20	0	40416	STA	0	FLOAD
	33	0	40413	FDV	0	JLOADVAL
40465	20	0	40423	STA	0	B
	12	0	40416	LDA	0	FLOAD
40466	32	0	40417	FMU	0	RHOSQ
	20	0	40415	STA	0	FMOTOR
40467	12	0	40404	LDA	0	A
	32	0	40414	FMU	0	TOTINERT
40470	31	0	40415	FSB	0	FMOTOR
	20	0	40415	STA	0	FMOTOR
40471	33	0	40412	FDV	0	MOINERT
	20	0	40422	STA	0	D
40472	54	3	00001	ISK	3	1
	75	0	40043	SLJ	0	NEWINPUT+1
40473	54	4	00000	ISK	4	0
	75	0	40043	SLJ	0	NEWINPUT+1
40474	54	6	00005	ISK	6	5
	75	0	40043	SLJ	0	NEWINPUT+1
40475	54	5	00004	ISK	5	4
	75	0	40043	SLJ	0	NEWINPUT+1
40476	12	0	40564	LDA	0	STOPINDX
	20	0	40563	STA	0	STOPNOW
40477	75	0	40043	SLJ	0	NEWINPUT+1
	50	0	00000	ENI	0	0
40500	12	0	40512	LDA	0	NOFLOAT
	20	0	40071	STA	0	FLOATEST
40501	12	0	40404	LDA	0	A
	20	0	40514	STA	0	PRINDELT
40502	12	0	40414	LDA	0	TOTINERT
	20	0	40515	STA	0	PRINDELT+1
40503	12	0	40420	LDA	0	KMOTCONS
	20	0	40516	STA	0	PRINDELT+2
40504	12	0	40424	LDA	0	OMEGANSQ
	20	0	40517	STA	0	PRINDELT+3
40505	12	0	40411	LDA	0	RHO
	20	0	40520	STA	0	PRINDELT+4
40506	12	0	40417	LDA	0	RHOSQ
	20	0	40521	STA	0	PRINDELT+5
40507	72	0	40510	RAO	0	/+1
	75	4	71000	SLJ	4	DECOF
40510	01	0	40514	01	0	PRINDELT
	06	0	00000	06	0	0
40511	72	0	40046	RAO	0	NEWINPUT+4
	75	0	40053	SLJ	0	COMBINED
40512	75	0	40053	SLJ	0	COMBINED
	50	0	00000	ENI	0	0
40513	13	0	40162	LAC	0	THETA
	30	0	40664	FAD	0	ONE
40514	00	0	00000	BSS	6	
	00	0	00000			
40522	75	3	40523	LIMITCYC	3	/+1
	75	0	40525	SLJ	0	/+3
40523	50	0	00000	ENI	0	0
	12	0	00000	LDA	0	0
40524	65	0	40603	THS	0	TIME90SC
	75	0	40566	SLJ	0	TIMESTOP
40525	12	0	40350	LDA	0	PRINTBUF+2
	31	0	40664	FSB	0	ONE
40526	22	3	40534	AJP	3	PRINTNOW
	50	2	77775	ENI	2	77775

LINEAR

NOFLOAT

NOSINK

PRINDELT

LIMITCYC

40527	12	0	40607	LDA	0	LIMITBUF+2
	31	0	40350	FSB	0	PRINTBUF+2
40530	22	2	40360	AJP	2	EXIT
	12	0	40346	LDA	0	PRINTBUF
40531	20	0	40605	STA	0	LIMITBUF
	12	0	40347	LDA	0	PRINTBUF+1
40532	20	0	40606	STA	0	LIMITBUF+1
	12	0	40350	LDA	0	PRINTBUF+2
40533	20	0	40607	STA	0	LIMITBUF+2
	75	0	40360	SLJ	0	EXIT
40534	54	2	77775	PRINTNOW ISK	2	77775
	75	0	40360	SLJ	0	EXIT
40535	12	0	40615	LDA	0	INDEXAVE
	15	0	40617	SUB	0	COUNTZET
40536	22	0	40541	AJP	0	/+3
	14	0	40617	ADD	0	COUNTZET
40537	20	0	40615	STA	0	INDEXAVE
	72	0	40615	RAO	0	INDEXAVE
40540	75	0	40543	SLJ	0	LIMPRINT
	50	0	00000	ENI	0	0
40541	12	0	40607	LDA	0	LIMITBUF+2
	30	0	40613	FAD	0	LIMITBUF+6
40542	20	0	40613	STA	0	LIMITBUF+6
	50	0	00000	ENI	0	0
40543	74	0	02000	LIMPRINT EXF	0	02000
	75	4	71000	SLJ	4	DECOF
40544	01	0	40605	01	0	LIMITBUF
	06	0	00001	06	0	1
40545	72	0	40544	RAO	0	/-1
	72	0	40614	RAO	0	LIMINDEX
40546	12	0	40614	LDA	0	LIMINDEX
	15	0	40616	SUB	0	STOPZETA
40547	22	0	40555	AJP	0	STOPAVE
	14	0	40616	ADD	0	STOPZETA
40550	20	0	40614	STA	0	LIMINDEX
	12	0	40604	LDA	0	TOOSMALL
40551	31	0	40607	FSB	0	LIMITBUF+2
	22	2	40573	AJP	2	SMALTIME
40552	10	0	00000	ENA	0	0
	20	0	40607	STA	0	LIMITBUF+2
40553	20	0	00000	STA	0	0
	74	0	01000	EXF	0	01000
40554	75	0	40360	SLJ	0	EXIT
	50	0	00000	ENI	0	0
40555	10	0	00000	STOPAVE ENA	0	0
	20	0	40605	STA	0	LIMITBUF
40556	20	0	40606	STA	0	LIMITBUF+1
	20	0	40607	STA	0	LIMITBUF+2
40557	12	0	40613	LDA	0	LIMITBUF+6
	33	0	40620	FDV	0	DIVIDE
40560	31	0	40664	FSB	0	ONE
	20	0	40610	STA	0	LIMITBUF+3
40561	75	4	71000	SLJ	4	DECOF
	50	0	00000	ENI	0	0
40562	01	0	40605	STOPPRINT 01	0	LIMITBUF
	06	0	00000	06	0	0
40563	76	1	40022	STOPNOW SLS	1	GOAROUND
	50	0	00000	ENI	0	0
40564	76	0	40000	STOPINDX SLS	0	START
	50	0	00000	ENI	0	0
40565	76	1	40022	NEWSTOP SLS	1	GOAROUND
	50	0	00000	ENI	0	0
40566	50	2	00000	TIMESTOP ENI	2	0
	75	4	71000	SLJ	4	DECOF
40567	01	0	40605	01	0	LIMITBUF
	04	0	01570	04	0	1570
40570	74	0	02000	EXF	0	02000
	75	4	71000	SLJ	4	DECOF
40571	01	0	40346	01	0	PRINTBUF
	04	0	01747	04	0	1747
40572	75	0	40563	SLJ	0	STOPNOW
	50	0	00000	ENI	0	0
40573	75	3	40574	SMALTIME SLJ	3	/+1
	75	0	40552	SLJ	0	LIMPRINT+7



40574	50 0 00000	ENI	0 0
	12 0 00000	LDA	0 0
40575	65 0 40602	THS	0 TIME30SC
	75 0 40577	SLJ	0 /+2
40576	75 0 40552	SLJ	0 LIMPRINT+7
	50 0 00000	ENI	0 0
40577	75 4 71000	SLJ	4 DECOF
	50 0 00000	ENI	0 0
40600	01 0 40346	01	0 PRINTBUF
	04 0 01755	04	0 1755
40601	75 0 40563	SLJ	0 STOPNOW
	50 0 00000	ENI	0 0
40602	00 0 00000	TIME30SC	OCT 3410
	00 0 03410		
40603	00 0 00000	TIME90SC	OCT 12430
	00 0 12430		
40604	20 0 14012	TOOSMALL	DEC 1.005
	17 2 70243		
40605	00 0 00000	LIMITBUF	BSS 7
	00 0 00000		
40614	00 0 00000	LIMINDEX	BSS 1
	00 0 00000		
40615	00 0 00000	INDEXAVE	BSS 1
	00 0 00000		
40616	00 0 00000	STOPZETA	BSS 1
	00 0 00000		
40617	00 0 00000	COUNTZET	BSS 1
	00 0 00000		
40620	00 0 00000	DIVIDE	BSS 1
	00 0 00000		
40621	00 0 40346	PRINTSET	00 0 PRINTBUF
	06 0 00001	06	0 1
40622	01 0 40605	NEWLIMPT	01 0 LIMITBUF
	06 0 00001	06	0 1
40623	17 7 36314	ZETAINDX	DEC 0.05
	63 1 46314		
40624	17 7 46314		DEC 0.1
	63 1 46314		
40625	17 7 56314		DEC 0.2
	63 1 46314		
40626	17 7 64631		DEC 0.3
	46 3 14631		
40627	17 7 66314		DEC 0.4
	63 1 46314		
40630	20 0 04000		DEC 0.5
	00 0 00000		
40631	20 0 04631		DEC 0.6
	46 3 14631		
40632	20 0 05463		DEC 0.7
	14 6 31463		
40633	20 0 06314		DEC 0.8
	63 1 46314		
40634	20 0 07146		DEC 0.9
	31 4 63146		
40635	20 0 14000		DEC 1.0
	00 0 00000		
40636	20 0 06314	POINT8	DEC .8
	63 1 46314		
40637	20 0 04631	POINT6	DEC .6
	46 3 14631		
40640	20 0 04000	POINT5	DEC .5
	00 0 00000		
40641	17 7 66314	POINT4	DEC .4
	63 1 46314		
40642	17 7 64631	POINT3	DEC .3
	46 3 14631		
40643	17 7 56314	POINT2	DEC 0.2
	63 1 46314		
40644	17 7 46314	POINT1	DEC 0.1
	63 1 46314		
40645	00 0 00000	STOP6	OCT 6
	00 0 00006		
40646	00 0 00000	STOP7	OCT 7
	00 0 00007		

40647	00 0 00000	STOP8	OCT	10
	00 0 00010			
40650	00 0 00000	STOP10	OCT	12
	00 0 00012			
40651	00 0 00000	STOP14	OCT	16
	00 0 00016			
40652	00 0 00000	STOP20	OCT	24
	00 0 00024			
40653	00 0 00000	STOP17	OCT	21
	00 0 00021			
40654	00 0 00000	STOP25	OCT	31
	00 0 00031			
40655	00 0 00000	COUNT4	OCT	4
	00 0 00004			
40656	00 0 00000	COUNT5	OCT	5
	00 0 00005			
40657	00 0 00000		OCT	5
	00 0 00005			
40660	00 0 00000	COUNT7	OCT	7
	00 0 00007			
40661	00 0 00000	COUNT10	OCT	12
	00 0 00012			
40662	00 0 00000	COUNT12	OCT	14
	00 0 00014			
40663	00 0 00000		OCT	14
	00 0 00014			
40664	20 0 14000	ONE	DEC	1.0
	00 0 00000			
40665	20 0 24000	TWO	DEC	2.0
	00 0 00000			
40666	20 0 24000		DEC	2.0
	00 0 00000			
40667	20 0 26000	THREE	DEC	3.0
	00 0 00000			
40670	20 0 26000		DEC	3.0
	00 0 00000			
40671	20 0 34000	FOUR	DEC	4.0
	00 0 00000			
40672	20 0 44000	EIGHT	DEC	8.0
	00 0 00000			
40673	20 0 35000	FIVE	DEC	5.0
	00 0 00000			
40674	50 1 00000	MANUAL	ENI	1 0
	75 4 71000		SLJ	4 DECOF
40675	01 0 40605		01	0 LIMITBUF
	04 0 01570		04	0 1570
40676	50 2 00000		ENI	2 0
	75 4 71000		SLJ	4 DECOF
40677	01 0 40346		01	0 PRINTBUF
	04 0 03651		04	0 3651
40700	75 0 40563		SLJ	0 STOPNOW
	50 0 00000		ENI	0 0
40701	50 1 00000	FAULPRIN	ENI	1 0
	75 4 71000		SLJ	4 DECOF
40702	01 0 40605		01	0 LIMITBUF
	04 0 01570		04	0 1570
40703	50 2 00000		ENI	2 0
	75 4 71000		SLJ	4 DECOF
40704	01 0 40346		01	0 PRINTBUF
	04 0 01232		04	0 1232
40705	74 0 42003		EXF	0 42003
	76 0 40000		SLS	0 START
40706	20 0 14000	RESTVALU	DEC	1.0
	00 0 00000			
40707	00 0 00000		DEC	0
	00 0 00000			
40710	20 0 04631		DEC	0.6
	46 3 14631			
40711	20 0 06314		DEC	0.8
	63 1 46314			
40712	00 0 00000		BSS	14
	00 0 00000			
40726	17 7 64631	DELTAVAL	DEC	.3
	46 3 14631			



40727	17 7 46314	DEC	.1
	63 1 46314		
40730	17 7 27534	DEC	.03
	12 1 72702		
40731	17 7 15075	DEC	0.01
	34 1 21727		
40732	17 7 05075	DEC	0.005
	34 1 21727		
40733	00 0 00000	BSS	14
	00 0 00000		
40747	17 7 46314	JMOTVALU DEC	.1
	63 1 46314		
40750	17 7 56314	DEC	.2
	63 1 46314		
40751	20 0 04000	DEC	.5
	00 0 00000		
40752	20 0 06314	DEC	.8
	63 1 46314		
40753	20 0 07146	DEC	.9
	31 4 63146		
40754	00 0 00000	BSS	20
	00 0 00000		
40774	00 0 00000	FLFTRATO DEC	0
	00 0 00000		
40775	17 7 56314	DEC	.2
	63 1 46314		
40776	17 7 66314	DEC	.4
	63 1 46314		
40777	20 0 04631	DEC	.6
	46 3 14631		
41000	20 0 06314	DEC	.8
	63 1 46314		
41001	20 0 14000	DEC	1.0
	00 0 00000		
41002	17 7 35075	DEC	0.04
	34 1 21727		
41003	17 7 46314	DEC	0.1
	63 1 46314		
41004	20 0 07146	DEC	0.9
	31 4 63146		
41005	00 0 00000	BSS	20
	00 0 00000		
		END	

Solution 3 number	2S <sub>W</sub> n 99999999	J <sub>L</sub> 99999999	J <sub>m</sub> 99999999	e 99999999	Δ 29999999	ρ 99999999
point number	J <sub>m</sub> 49999999	J <sub>L</sub> 49999999	J <sub>m</sub> 99999999	F <sub>m</sub> 20000000	F <sub>L</sub> 79999999	ρ <sup>2</sup> 99999999
1	K 99999999	19999999	1	40000000	15999999	1
2	10000000	95004083-	1	48334152-	1	1
3	19999999	18006400	1	18669244-	1	1
4	29999999	25531729	1	40519239-	1	1
5	40000000	32098164	1	69412993-	1	1
6	50000000	37734520	1	10440547	1	1
7	59999999	42475713	1	14458358	1	1
8	69999999	46361850	1	18907183	1	1
9	79999999	49437347	1	23703703	1	1
10	89999999	51750106	1	28769225	1	1
11	99999999	53350719	1	34029984	1	1
12	10999999	54291737	1	39417381	1	1
13	11999999	54626982	1	44868139	1	1
14	12999999	54410913	1	50324400	1	1
15	14000000	53698048	1	55733762	1	1
16	15000000	52542443	1	61049253	1	1
17	16000000	50997217	1	66229265	1	1
18	17000000	49114140	1	71237437	1	1
19	18000000	46943268	1	76042502	1	1
20	19000000	44532629	1	80618101	1	1
21	20000000	41927962	1	84942563	1	1
22	21000000	39172497	1	88998670	1	1
23	22000000	36306785	1	92773391	1	1
24	23000000	33368563	1	96257613	1	1
25	24000000	30392671	1	99445851	1	1
26	25000000	27410989	1	10233595	1	1
27	26000000	24452419	1	10492882	1	1
28	27000000	21542892	1	10722807	1	1
29	28000000	18705407	1	10923980	1	1
30	29000000	15989086	1	11097264	1	1
31	30000000	13625000	1	11245019	1	1
32	31000000	11610459	1	11370928	1	1
33	32000000	98937811-	1	11478221	1	1
34	33000000	84309241-	1	11569649	1	1
35	34000000	71843596-	1	11647559	1	1
36	35000000	61221074-	1	11713950	1	1
37	36000000	52169158-	1	11770525	1	1
38	37000000	44455624-	1	11818734	1	1
39	38000000	37882584-	1	11859816	1	1
40	39000000	32281408-	1	11894823	1	1
41	40000000	27508401-	1	11924654	1	1
42	41000000	23441113-	1	11950075	1	1
43	42000000	19975199-	1	11971737	1	1
44	43000000	17021742-	1	11990196	1	1
45	44000000	14504971-	1	12005926	1	1
46	45000000	12360321-	1	12019330	1	1
47	46000000	10532771-	1	12030752	1	1
48	46600000	95686438-	2	12036778	1	1
49	46600000	-37900604	2	12036778	1	1
50	47000000	-35550956	2	11889925	1	1
51	48000000	-30294526	2	11561398	1	1
52	49000000	-25815292	2	11281446	1	1
53	50000000	-21998341	2	11042886	1	1
54	51000000	-18745749	2	10839599	1	1
55	52000000	-15974074	2	10666370	1	1
56	53000000	-13612208	2	10518753	1	1
57	54000000	-113612208	2	10366778	1	1
58	55000000	-906605683-	2	102036778	1	1
59	56000000	-60605683-	2	10036778	1	1
60	57000000	-39333994-	2	986605683-	1	1
61	58000000	-25815292	2	969208537	1	1
62	59000000	-21998341	2	95208537	1	1
63	60000000	-18745749	2	935974404	1	1
64	61000000	-15974074	2	92471132	1	1
65	62000000	-13612208	2	90243442	1	1
66	63000000	-113612208	2	90243442	1	1
67	64000000	-906605683-	2	90374327	1	1
68	65000000	-60605683-	2	90140801	1	1
69	66000000	-39333994-	2	89611697	1	1
70	67000000	-25815292	2	88853284	1	1
71	68000000	-21998341	2	87921151	1	1
72	69000000	-18745749	2	86861815	1	1
73	70000000	-15974074	2	85714102	1	1
74	71000000	-13612208	2	84942563	1	1
75	72000000	-113612208	2	84942563	1	1
76	73000000	-906605683-	2	84942563	1	1
77	74000000	-60605683-	2	84942563	1	1
78	75000000	-39333994-	2	84942563	1	1
79	76000000	-25815292	2	84942563	1	1
80	77000000	-21998341	2	84942563	1	1
81	78000000	-18745749	2	84942563	1	1
82	79000000	-15974074	2	84942563	1	1
83	80000000	-13612208	2	84942563	1	1
84	81000000	-113612208	2	84942563	1	1
85	82000000	-906605683-	2	84942563	1	1
86	83000000	-60605683-	2	84942563	1	1
87	84000000	-39333994-	2	84942563	1	1
88	85000000	-25815292	2	84942563	1	1
89	86000000	-21998341	2	84942563	1	1
90	87000000	-18745749	2	84942563	1	1
91	88000000	-15974074	2	84942563	1	1
92	89000000	-13612208	2	84942563	1	1
93	90000000	-113612208	2	84942563	1	1
94	91000000	-906605683-	2	84942563	1	1
95	92000000	-60605683-	2	84942563	1	1
96	93000000	-39333994-	2	84942563	1	1
97	94000000	-25815292	2	84942563	1	1
98	95000000	-21998341	2	84942563	1	1
99	96000000	-18745749	2	84942563	1	1
100	97000000	-15974074	2	84942563	1	1
101	98000000	-13612208	2	84942563	1	1
102	99000000	-113612208	2	84942563	1	1
103	100000000	-906605683-	2	84942563	1	1
104	101000000	-60605683-	2	84942563	1	1
105	102000000	-39333994-	2	84942563	1	1
106	103000000	-25815292	2	84942563	1	1
107	104000000	-21998341	2	84942563	1	1
108	105000000	-18745749	2	84942563	1	1
109	106000000	-15974074	2	84942563	1	1
110	107000000	-13612208	2	84942563	1	1
111	108000000	-113612208	2	84942563	1	1
112	109000000	-906605683-	2	84942563	1	1
113	110000000	-60605683-	2	84942563	1	1
114	111000000	-39333994-	2	84942563	1	1
115	112000000	-25815292	2	84942563	1	1
116	113000000	-21998341	2	84942563	1	1
117	114000000	-18745749	2	84942563	1	1
118	115000000	-15974074	2	84942563	1	1
119	116000000	-13612208	2	84942563	1	1
120	117000000	-113612208	2	84942563	1	1
121	118000000	-906605683-	2	84942563	1	1
122	119000000	-60605683-	2	84942563	1	1
123	120000000	-39333994-	2	84942563	1	1
124	121000000	-25815292	2	84942563	1	1
125	122000000	-21998341	2	84942563	1	1
126	123000000	-18745749	2	84942563	1	1
127	124000000	-15974074	2	84942563	1	1
128	125000000	-13612208	2	84942563	1	1
129	126000000	-113612208	2	84942563	1	1
130	127000000	-906605683-	2	84942563	1	1
131	128000000	-60605683-	2	84942563	1	1
132	129000000	-39333994-	2	84942563	1	1
133	130000000	-25815292	2	84942563	1	1
134	131000000	-21998341	2	84942563	1	1
135	132000000	-18745749	2	84942563	1	1
136	133000000	-15974074	2	84942563	1	1
137	134000000	-13612208	2	84942563	1	1
138	135000000	-113612208	2	84942563	1	1
139	136000000	-906605683-	2	84942563	1	1
140	137000000	-60605683-	2	84942563	1	1
141	138000000	-39333994-	2	84942563	1	1
142	139000000	-25815292	2	84942563	1	1
143	140000000	-21998341	2	84942563	1	1
144	141000000	-18745749	2	84942563	1	1
145	142000000	-15974074	2	84942563	1	1
146	143000000	-13612208	2	84942563	1	1
147	144000000	-113612208	2	84942563	1	1
148	145000000	-906605683-	2	84942563	1	1
149	146000000	-60605683-	2	84942563	1	1
150	147000000	-39333994-	2	84942563	1	1
151	148000000	-25815292	2	84942563	1	1
152	149000000	-21998341	2	84942563	1	1
153	150000000	-18745749	2	84942563	1	1
154	151000000	-15974074	2	84942563	1	1
155	152000000	-13612208	2	84942563	1	1
156	153000000	-113612208	2	84942563	1	1
157	154000000	-906605683-	2	84942563	1	1
158	155000000	-60605683-	2	84942563	1	1
159	156000000	-39333994-	2	84942563	1	1
160	157000000	-25815292	2	84942563	1	1
161	158000000	-21998341	2	84942563	1	1
162	159000000	-18745749	2	84942563	1	1
163	160000000	-15974074	2	84942563	1	1
164	161000000	-13612208	2	84942563	1	1
165	162000000	-113612208	2	84942563	1	1
166	163000000	-906605683-	2	84942563	1	1
167	164000000	-60605683-	2	84942563	1	1
168	165000000	-39333994-	2	84942563	1	1
169	166000000	-25815292	2	84942563	1	1
170	167000000	-21998341	2	84942563	1	1



Solution 1	280n79999999	Jm/JL 11111111	fL/fT 39999999	C 99999999	Δ 29999999	ρ 99999999
number	Jm 99999999-	1 JL 89999999	Jmtp <sup>2</sup> JL 99999999	Fm 47999999	FL 31999999	ρ <sup>2</sup> 99999999
	K 99999999	K/Jm 99999999	1 fm/Jm 47999999	1 FL/JL 35555555	ω <sub>n</sub> <sup>2</sup> 99999999	ξ 39999999
number 1	t 33800000	1 θ <sub>c</sub> 52456182-	1 θ <sub>c</sub> 13396964	1		
of maximum	2 11230000	2 38509278-	1 11053498	1		
	3 21289999	2 12863485-	1 10697107	1		
	4 32499999	2 79971808-	2 10583527	1		
	5 44279999	2 65238396-	2 10542905	1		
	6 56369999	2 59130680-	2 10524201	1		
	7 68649998	2 55271237-	2 10511522	1		
	8 81049998	2 53208045-	2 10504778	1		
	9 93519998	2 51991503-	2 10500457	1		
10	10602999	3 51279129-	2 10498432	1		
3	number of maximums averaged			50122288-	1	average positive error in limit cycle

# APPENDIX D 1

TYPICAL PRINT OUT ILLUSTRATING LIMIT CYCLE  
(Numbers indicated in tenths followed by power of 10)

2 2547999999  $J_m/J_L$  11111111  $f_L/F_T$  59999999  $E$  99999999  $\Delta$  29999999  $\rho$  99999999  
 $J_m$  99999999- 1  $J_L$  89999999  $J_m \rho^2 J_L$  99999999  $f_m$  32000000  $f_L$  47999999  $\rho^2$  99999999  
 $K$  99999999  $K/J_m$  99999999 1  $f_m/J_m$  32000000 1  $f_L/J_L$  53333333  $\omega_n^2$  99999999  $\xi$  39999999  
 number 1  $t$  35400000 1  $\dot{\Theta}_c$  17098208- 1  $\Theta_c$  13104190 1  
 of maximum 2 11580000 2 17034145- 1 10721242 1  
 3 23279999 2 18292628- 2 10323347 1  
 4 38859999 2 35712402- 3 10200613 1  
 5 58729999 2 72447348- 4 10138780 1  
 6 83939998 2 11243587- 4 10098946 1  
 7 11619999 3 10685365- 5 10071240 1  
 8 15810000 3 47854719- 7 10051391 1  
 9 21077001 3 29068282- 8 10037079 1  
 1005 215160011794 3 -167815371770 - 2 999996862854

1005 indicates  $\Theta_c$  max. less than 1005 and 30 sec. of  
 real time has elapsed since the last maximum  
 was recorded (low velocity).

The time velocity, and position of the last computed  
 point is printed.

#### APPENDIX D 2

TYPICAL PRINT OUT ILLUSTRATING NO LIMIT CYCLE  
 (Numbers indicated in tenths followed by power of 10)



thesA47

Steady state response of a second order



3 2768 001 91494 8

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